One-Way Analysis of variance

Elias Zintzaras, M.Sc., Ph.D.

Professor in Biomathematics-Biometry
Department of Biomathematics
School of Medicine
University of Thessaly

Institute for Clinical Research and Health Policy Studies
Tufts University School of Medicine
Boston, MA, USA

Theodoros Mprotsis, MSc, PhD
Teacher & Research Fellow
(http://biomath.med.uth.gr)
University of Thessaly
Email: tmprotsis@uth.gr

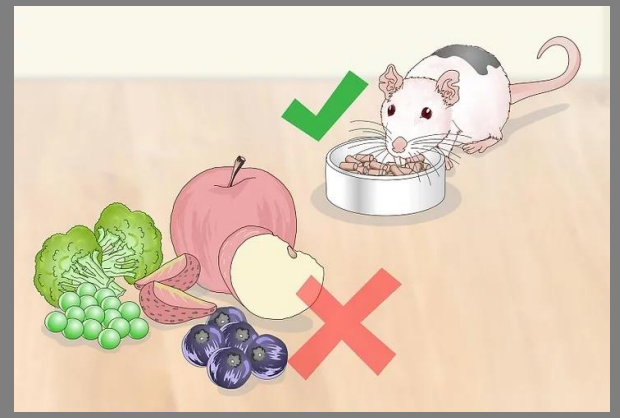
When;

- To determine if there is a statistically significant difference between the means of **two independent groups**, the **t-test for independent samples** is used
- In the case where there are more then two independent groups, the t-test for independent samples is not appropriate
- Therefore, to determine if there are any statistically significant differences between the means **of three or more** independent (unrelated) groups, an **analysis of variance (ANOVA)** should be used
- An alternative would be to apply the t-test for independent samples to every possible pair. However, when there are numerous groups, this method becomes challenging; for instance, if there were five groups, ten t-tests would be needed. Additionally, the possibility of error increases
- To determine which specific groups differed from each other, a **post hoc test** should be used

Assumptions

- The dependent variable is normally distributed in each group that is being compared (Shapiro-Wilk test for n<50 and Kolmogorov Smirnov for n>50)
- There is homogeneity of variances. This means that the population variances in each group are equal ($\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \ldots = \sigma_k^2 = \sigma^2$) (Levene's test)
- The groups are independent

Effect of different diets in mice



Effect of different diets in mice

In a study, the liver weight (expressed as a percentage of body weight) of mice from four groups, each fed a different diet, was recorded. We aim to investigate whether there are systematic differences between the four groups

	a	b	c	d
	3.42	3.17	3.34	3.64
	3.96	3.63	3.72	3.93
	3.87	3.38	3.81	3.77
	4.19	3.47	3.66	4.18
	3.58	3.39	3.55	4.21
	3.76	3.41	3.51	3.88
Mean	**3.80**	**3.41**	**3.60**	**3.94**

Data entry

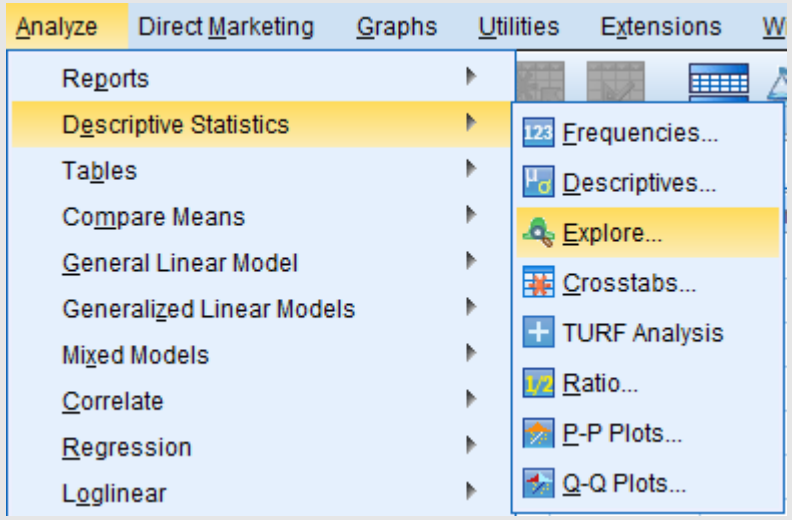
The numbers **1, 2, 3** and **4** in the variable group represent the diet groups **a, b, c** and **d**

	group	weight
1	1	3.42
2	1	3.96
3	1	3.87
4	1	4.19
5	1	3.58
6	1	3.76
7	2	3.17
8	2	3.63
9	2	3.38
10	2	3.47
11	2	3.39
12	2	3.41
13	3	3.34
14	3	3.72
15	3	3.81
16	3	3.66
17	3	3.55
18	3	3.51
19	4	3.64
20	4	3.93
21	4	3.77
22	4	4.18
23	4	4.21
24	4	3.88

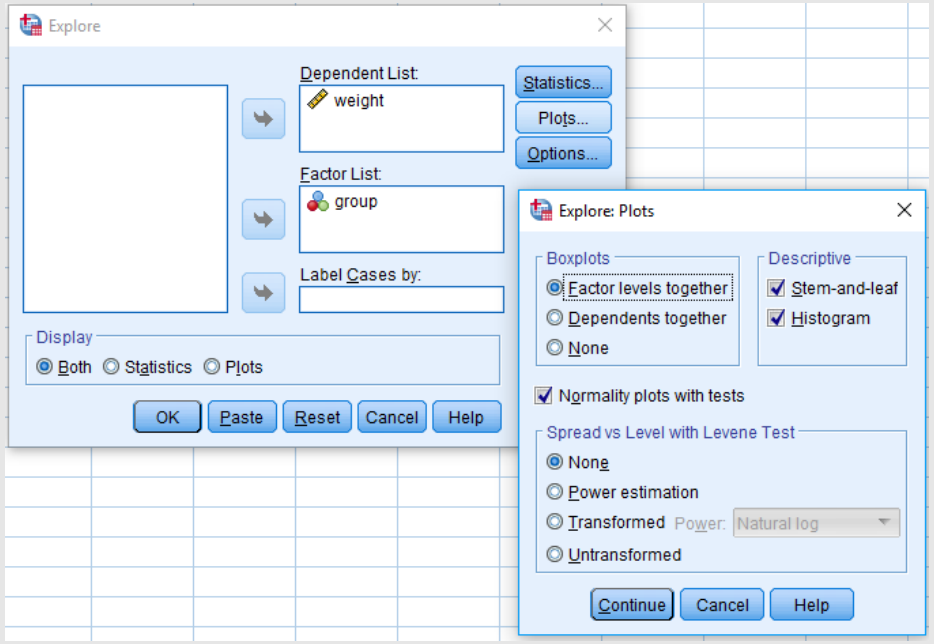
a	b	c	d
3.42	3.17	3.34	3.64
3.96	3.63	3.72	3.93
3.87	3.38	3.81	3.77
4.19	3.47	3.66	4.18
3.58	3.39	3.55	4.21
3.76	3.41	3.51	3.88

Test of normality

To test for normality, select **Analyze -> Descriptive Statistics -> Explore** from the menu.



Adjust the settings as shown below, then press **Continue** and **OK**



Test of normality

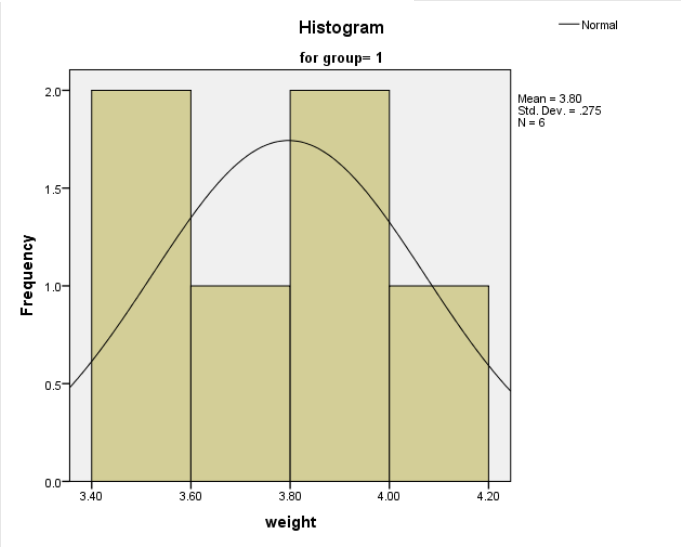
Tests of Normality

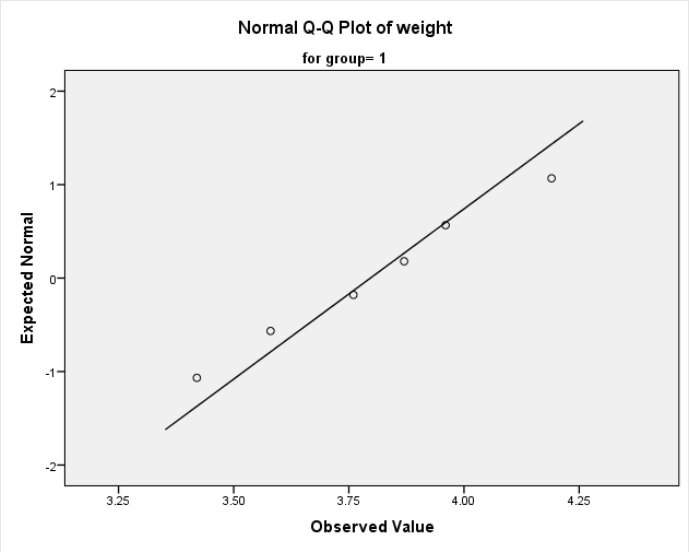
	group	Kolmogorov-Smirnov[a] Statistic	df	Sig.	Shapiro-Wilk Statistic	df	Sig.
weight	1	.118	6	.200*	.992	6	.993
	2	.258	6	.200*	.940	6	.657
	3	.144	6	.200*	.980	6	.953
	4	.195	6	.200*	.934	6	.609

*. This is a lower bound of the true significance.

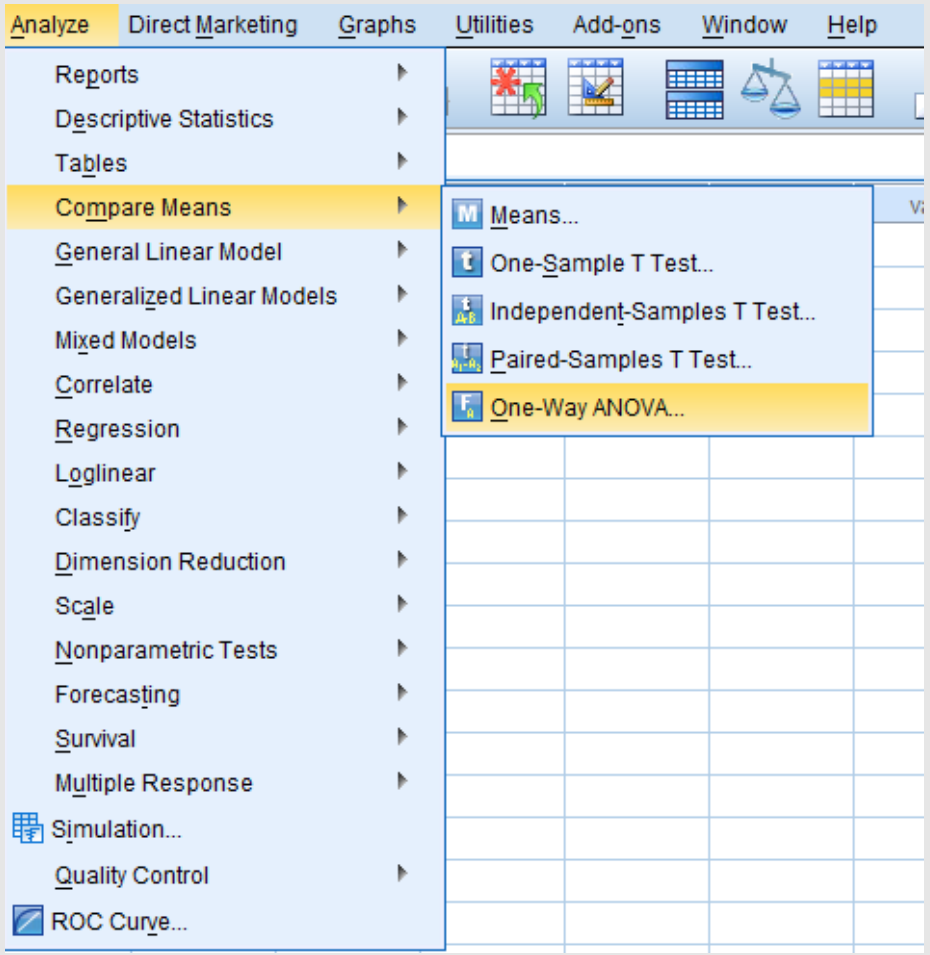
a. Lilliefors Significance Correction

In the **Test of Normality** table**,** the **Shapiro-Wilk test (n<50)** shows that the **Sig. value (p-value)** for all the categories of the qualitative variable (diet) is greater than 0.05. Thus, we do not reject the null hypothesis (H_0), indicating that the quantitative variable (weight) is **normally distributed** within each category of the nominal variable (diet)

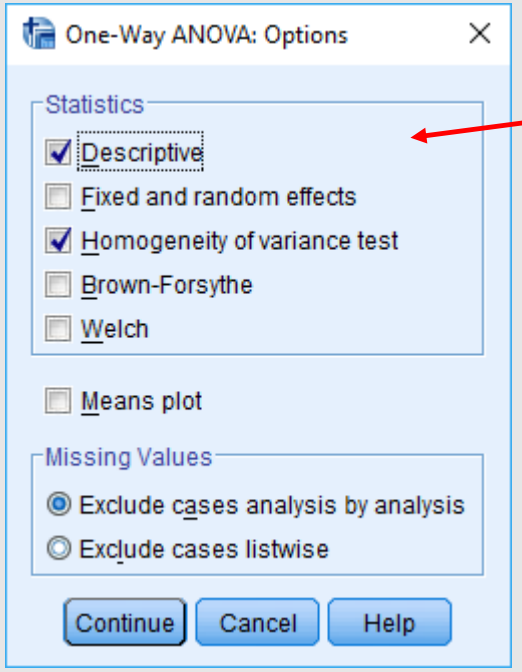


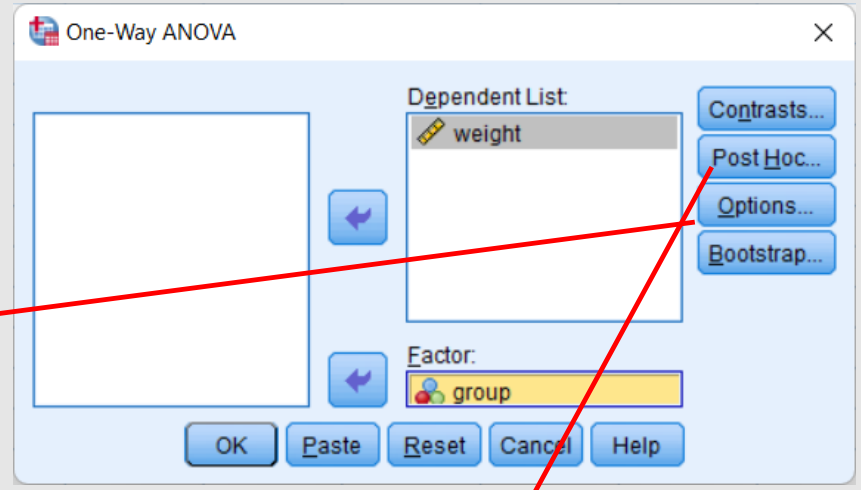


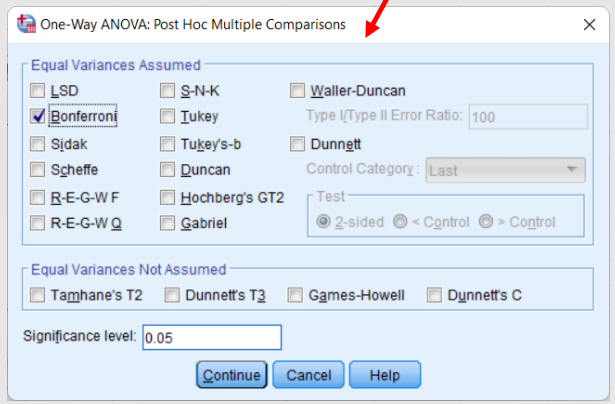
Running the one-way anova test



To analyze the data, choose **Analyze -> Compare Means -> One Way ANOVA** from the menu







Results and interpretation

Descriptives

weight

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1	6	3.7967	.27457	.11209	3.5085	4.0848	3.42	4.19
2	6	3.4083	.14892	.06080	3.2521	3.5646	3.17	3.63
3	6	3.5983	.16750	.06838	3.4226	3.7741	3.34	3.81
4	6	3.9350	.22492	.09182	3.6990	4.1710	3.64	4.21
Total	24	3.6846	.28247	.05766	3.5653	3.8039	3.17	4.21

Test of Homogeneity of Variances

weight

Levene Statistic	df1	df2	Sig.
1.076	3	20	.382

Levene's test indicates that the population variances are equal across the categories of the qualitative variable (diet), as the p-value is greater then 0.05 (the null hypothesis, H_0, is not rejected).

Null hypothesis (H_0): The variances of the dependent variable (weight) are equal across all groups

Results and interpretation

ANOVA

Weight

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,954	3	,318	7,220	,002
Within Groups	,881	20	,044		
Total	1,835	23			

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Weight

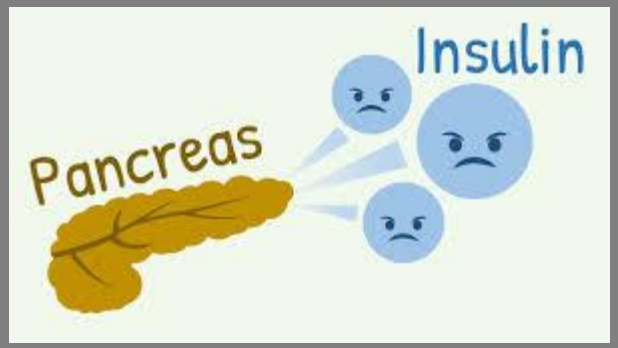
Bonferroni

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	,38833*	,12118	,027	,0336	,7430
	3	,19833	,12118	,704	-,1564	,5530
	4	-,13833	,12118	1,000	-,4930	,2164
2	1	-,38833*	,12118	,027	-,7430	-,0336
	3	-,19000	,12118	,795	-,5447	,1647
	4	-,52667*	,12118	,002	-,8814	-,1720
3	1	-,19833	,12118	,704	-,5530	,1564
	2	,19000	,12118	,795	-,1647	,5447
	4	-,33667	,12118	,070	-,6914	,0180
4	1	,13833	,12118	1,000	-,2164	,4930
	2	,52667*	,12118	,002	,1720	,8814
	3	,33667	,12118	,070	-,0180	,6914

*. The mean difference is significant at the 0.05 level.

- In the **ANOVA table**, the Sig. (p-value) for the between-group comparison is less than 0.05, indicating **a significant difference** between the diets
- The **Multiple Comparisons** table shows the individual comparisons between the groups with **Bonferroni correction**: The mean difference between groups 1 and 2 is **statistically significant** ($p<0.05$), with the 95% confidence interval (CI) for the difference ranging from 0.0336 to 0.7430, which does not include 0. There is also a statistically significant difference between groups 2 and 4

Insulin secretion of experimental animals



Insulin secretion of experimental animals

In an experiment insulin secretion was measured in pancreatic tissue samples of experimental animals. The samples were divided into 5 groups based on glucose levels. Each group corresponds to different glucose levels. We want to see if there is a statistically significant difference in the average values of these categories.

Group 1	1.53	1.69	3.75	2.89	3.26	2.83	2.86	2.59
Group 2	3.15	3.96	3.59	1.89	1.45	3.49	1.56	2.44
Group 3	3.89	4.80	3.68	5.70	5.62	5.79	4.75	5.33
Group 4	8.18	5.64	7.36	5.33	8.82	5.26	8.75	7.10
Group 5	5.86	5.46	5.69	6.49	7.81	9.03	7.49	8.98

- Group 1: Very low to low glucose levels (below 4.00)
- Group 2: Low to moderate glucose levels (1.45 to 3.96)
- Group 3: Moderate to high glucose levels (3.68 to 5.79)
- Group 4: High glucose levels with some extreme values (5.26 to 8.82)
- Group 5: High to very high glucose levels (5.46 to 9.03)

Data entry

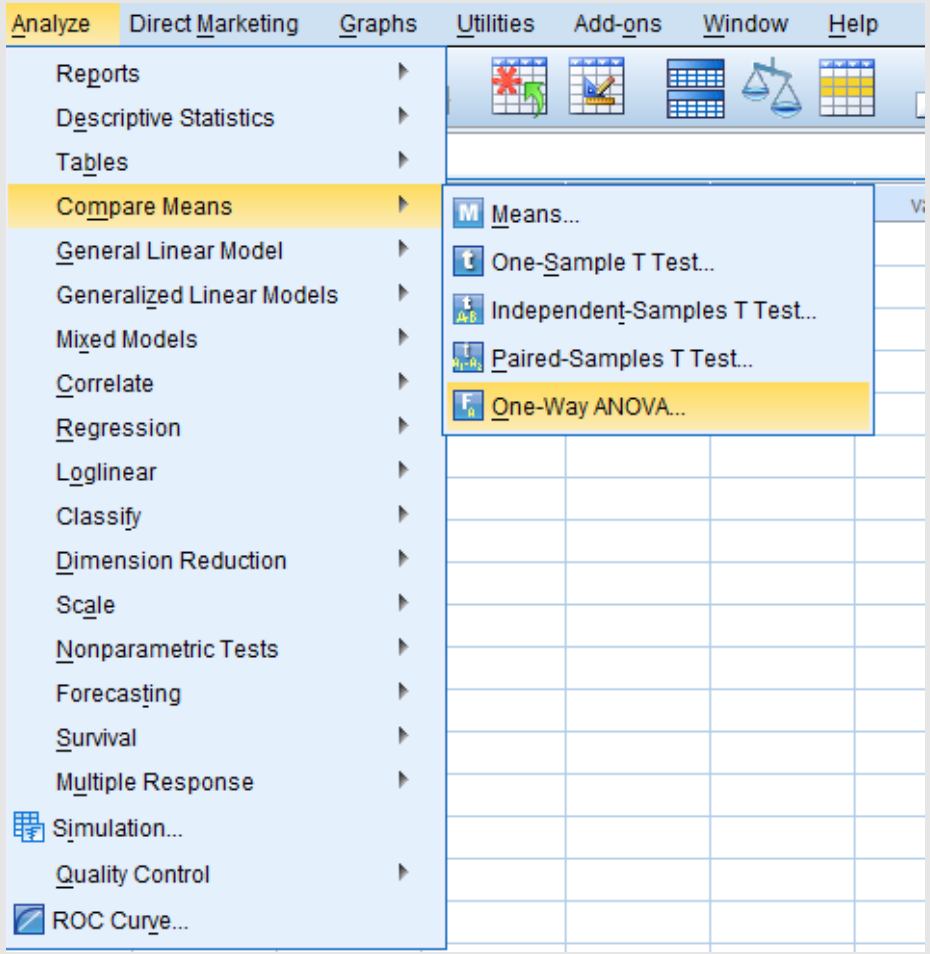
	group	level
1	1	1,53
2	1	1,69
3	1	3,75
4	1	2,89
5	1	3,26
6	1	2,83
7	1	2,86
8	1	2,59
9	2	3,15
10	2	3,96
11	2	3,59
12	2	1,89
13	2	1,45
14	2	3,49
15	2	1,56
16	2	2,44
17	3	3,89
18	3	4,80
19	3	3,68
20	3	5,70
21	3	5,62
22	3	5,79
23	3	4,75
24	3	5,33
25	4	8,18
26	4	5,64
27	4	7,36
28	4	5,33
29	4	8,82
30	4	5,26
31	4	8,75
32	4	7,10
33	5	5,86
34	5	5,46
35	5	5,69
36	5	6,49
37	5	7,81
38	5	9,03
39	5	7,49
40	5	8,98

1

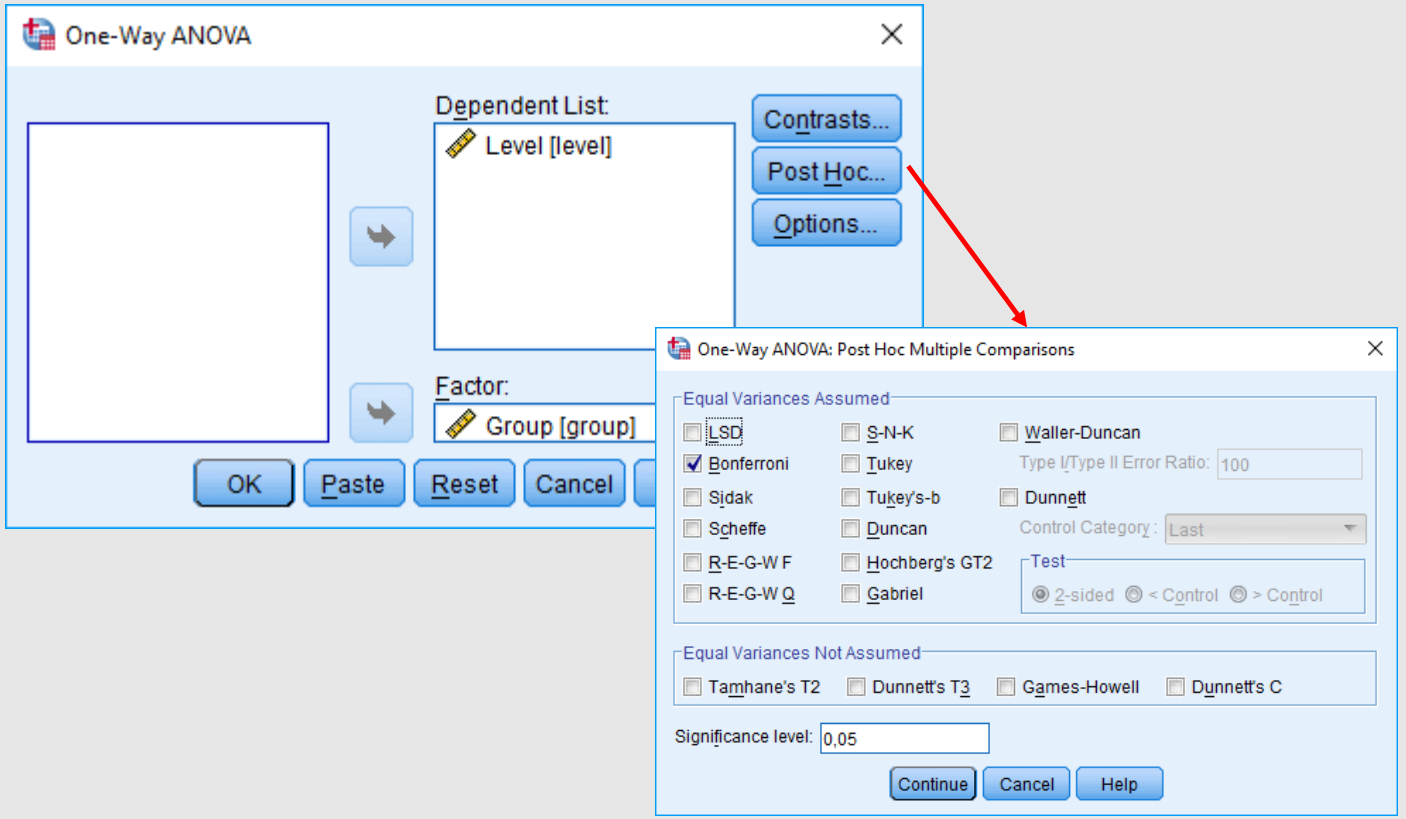
2

3

Running the One-Way ANOVA test



To analyze the data, choose **Analyze -> Compare Means -> One Way ANOVA** from the menu



Results and interpretation

ANOVA

Level

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	154,564	4	38,641	29,793	,000
Within Groups	45,394	35	1,297		
Total	199,959	39			

- In the **ANOVA table**, the Sig. (p-value) for the between-group comparison is less than 0.001, indicating **a significant difference** between the groups (group)

Test of Homogeneity of Variances

Level

Levene Statistic	df1	df2	Sig.
2,960	4	35	,033

- **Null hypothesis (H_0):** The variances of the dependent variable (level) are equal across all groups
- **Levene's test** indicates that the population variances are not equal across the categories of the qualitative variable, as the p-value is less then 0.05 (the null hypothesis, H_0, is rejected)

Post Hoc Test

Multiple Comparisons

Dependent Variable: Level

Bonferroni

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-,01625	,56942	1,000	-1,7223	1,6898
	3	-2,27000*	,56942	,003	-3,9760	-,5640
	4	-4,38000*	,56942	,000	-6,0860	-2,6740
	5	-4,42625*	,56942	,000	-6,1323	-2,7202
2	1	,01625	,56942	1,000	-1,6898	1,7223
	3	-2,25375*	,56942	,004	-3,9598	-,5477
	4	-4,36375*	,56942	,000	-6,0698	-2,6577
	5	-4,41000*	,56942	,000	-6,1160	-2,7040
3	1	2,27000*	,56942	,003	,5640	3,9760
	2	2,25375*	,56942	,004	,5477	3,9598
	4	-2,11000*	,56942	,007	-3,8160	-,4040
	5	-2,15625*	,56942	,006	-3,8623	-,4502
4	1	4,38000*	,56942	,000	2,6740	6,0860
	2	4,36375*	,56942	,000	2,6577	6,0698
	3	2,11000*	,56942	,007	,4040	3,8160
	5	-,04625	,56942	1,000	-1,7523	1,6598
5	1	4,42625*	,56942	,000	2,7202	6,1323
	2	4,41000*	,56942	,000	2,7040	6,1160
	3	2,15625*	,56942	,006	,4502	3,8623
	4	,04625	,56942	1,000	-1,6598	1,7523

*. The mean difference is significant at the 0.05 level.

The **Multiple Comparisons** table shows the individual comparisons between the groups with **Bonferroni correction**: The mean difference between groups 1 and 3 is **statistically significant** ($p<0.05$), with the 95% confidence interval (CI) for the difference ranging from -3.976 to -0.564, which does not include 0. There is also a statistically significant difference between groups 1 and 4, between groups 1 and 5, between groups 2 and 4, between groups 2 and 5, between groups 3 and 4, and between groups 3 and 5.

Practical exercise

Test whether a new analgesic medication (A) differs from an old medication (B) and from a placebo (C) in terms of time to relief. Twenty-one patients were randomly assigned to one of three groups: Group A (new medication), Group B (old medication), and Group C (placebo). The observed times to relief for each group were as follows:

ID	A	B	C
1	4.56	4.32	2.30
2	5.61	4.89	3.42
3	4.67	5.01	4.21
4	5.09	5.90	1.50
5	6.21	4.88	2.03
6	4.28	5.29	1.87
7	5.23	4.89	3.12