Normal distribution

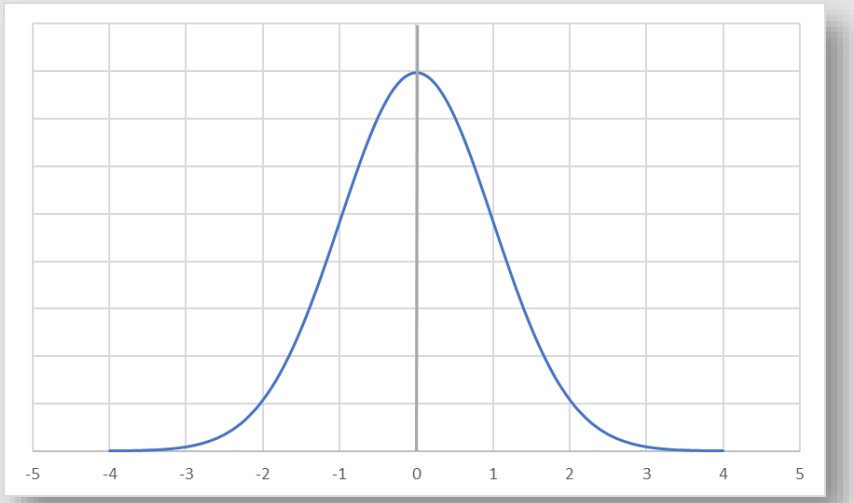
Elias Zintzaras, M.Sc., Ph.D.

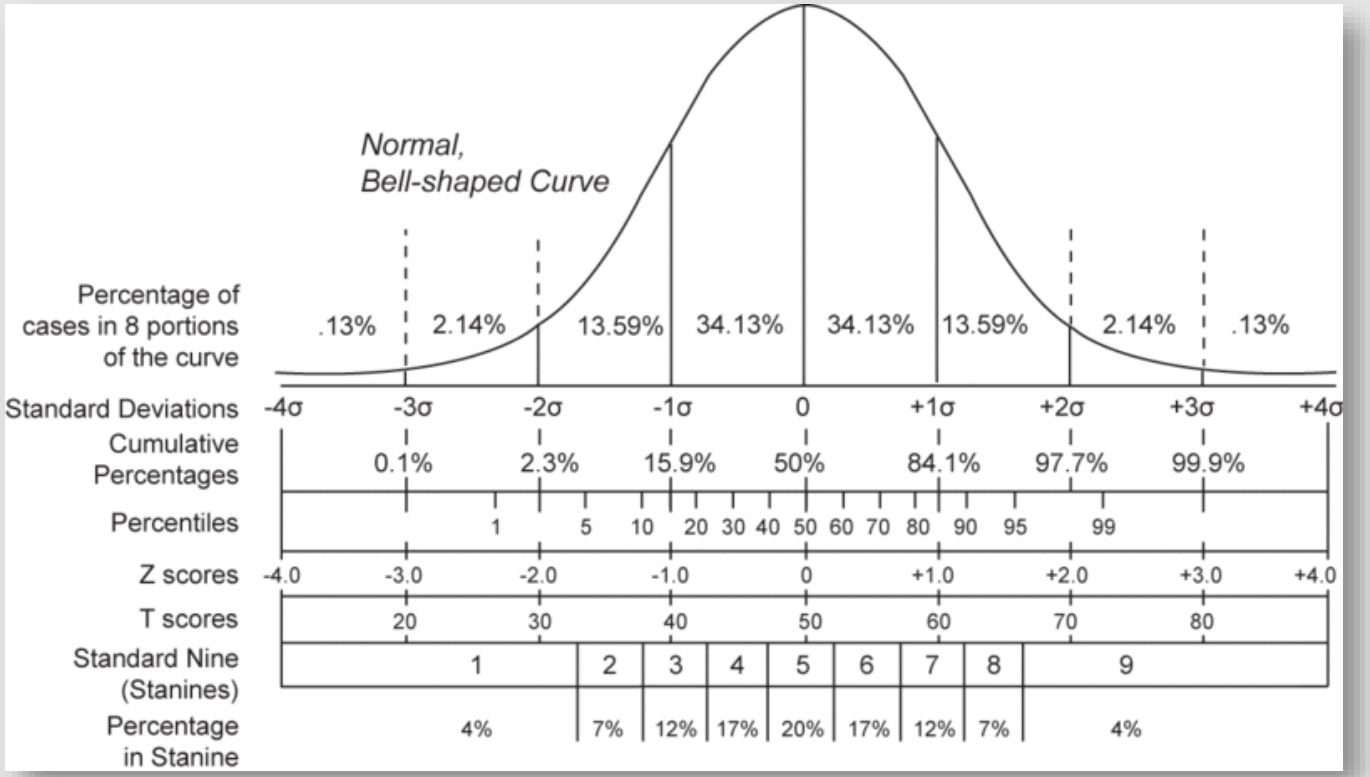
Professor in Biomathematics-Biometry
Department of Biomathematics
School of Medicine
University of Thessaly

Institute for Clinical Research and Health Policy Studies
Tufts University School of Medicine
Boston, MA, USA

Theodoros Mprotsis, MSc, PhD
Teacher & Research Fellow
(http://biomath.med.uth.gr)
University of Thessaly
Email: tmprotsis@uth.gr

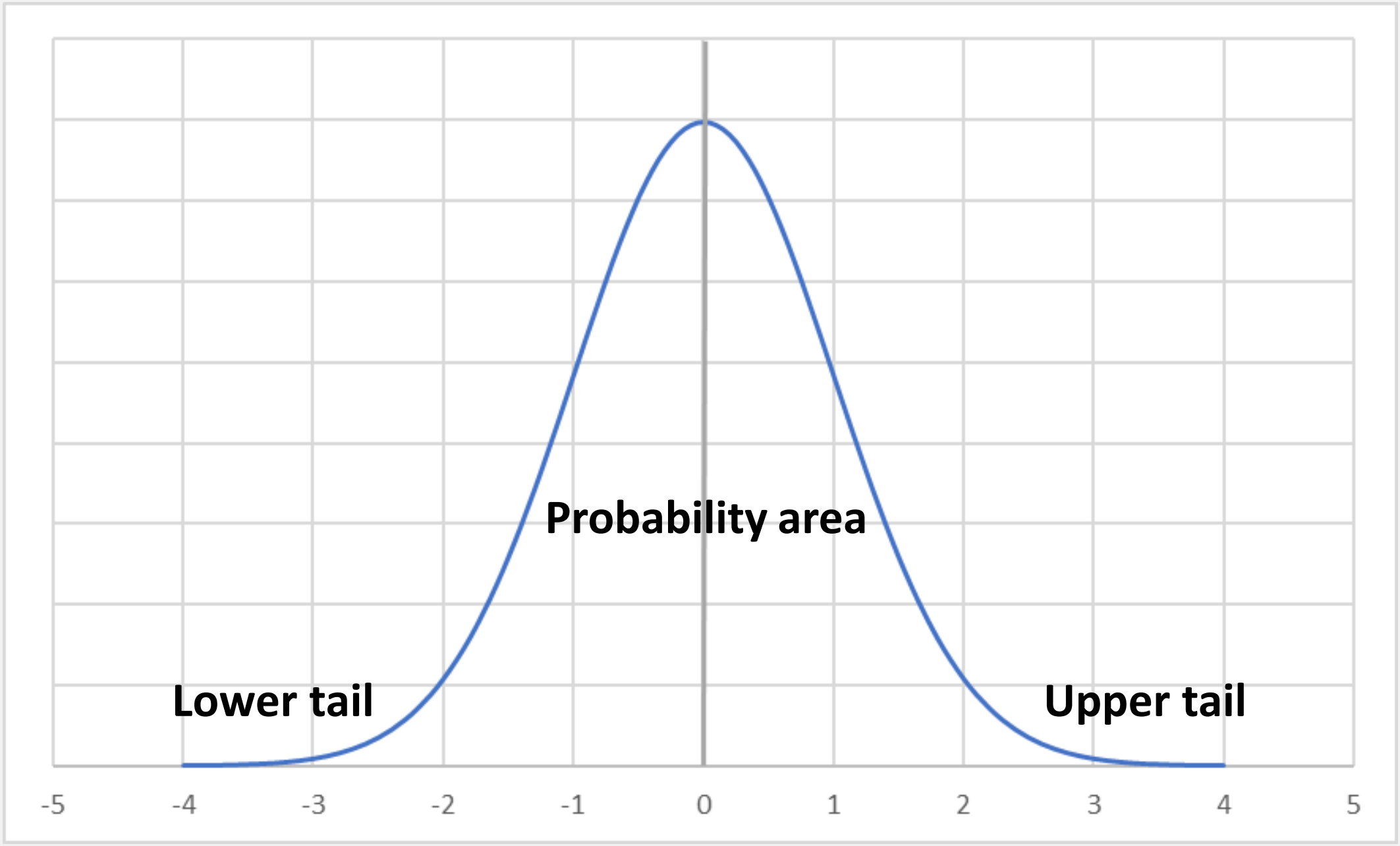
Have you seen this shape before?

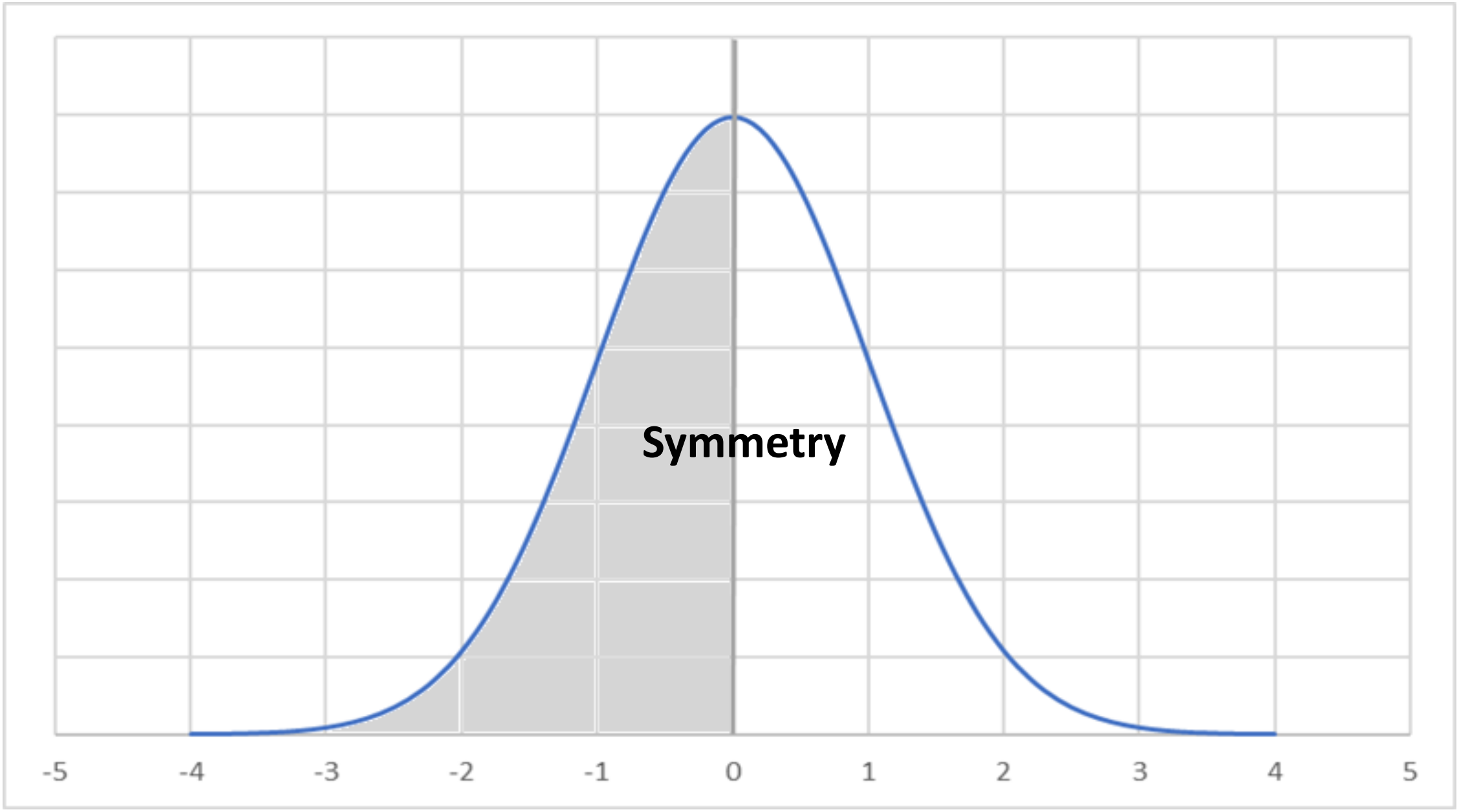


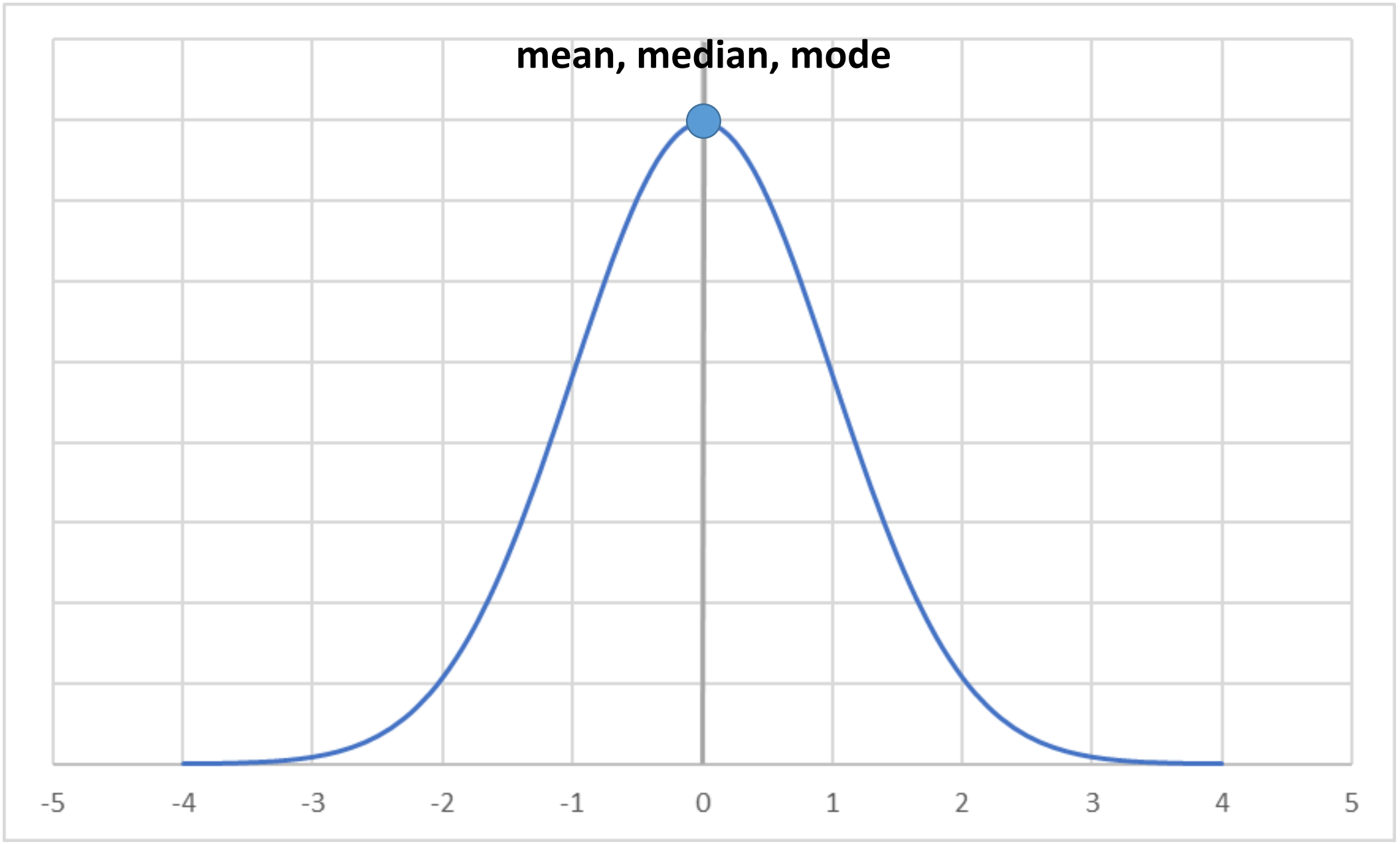


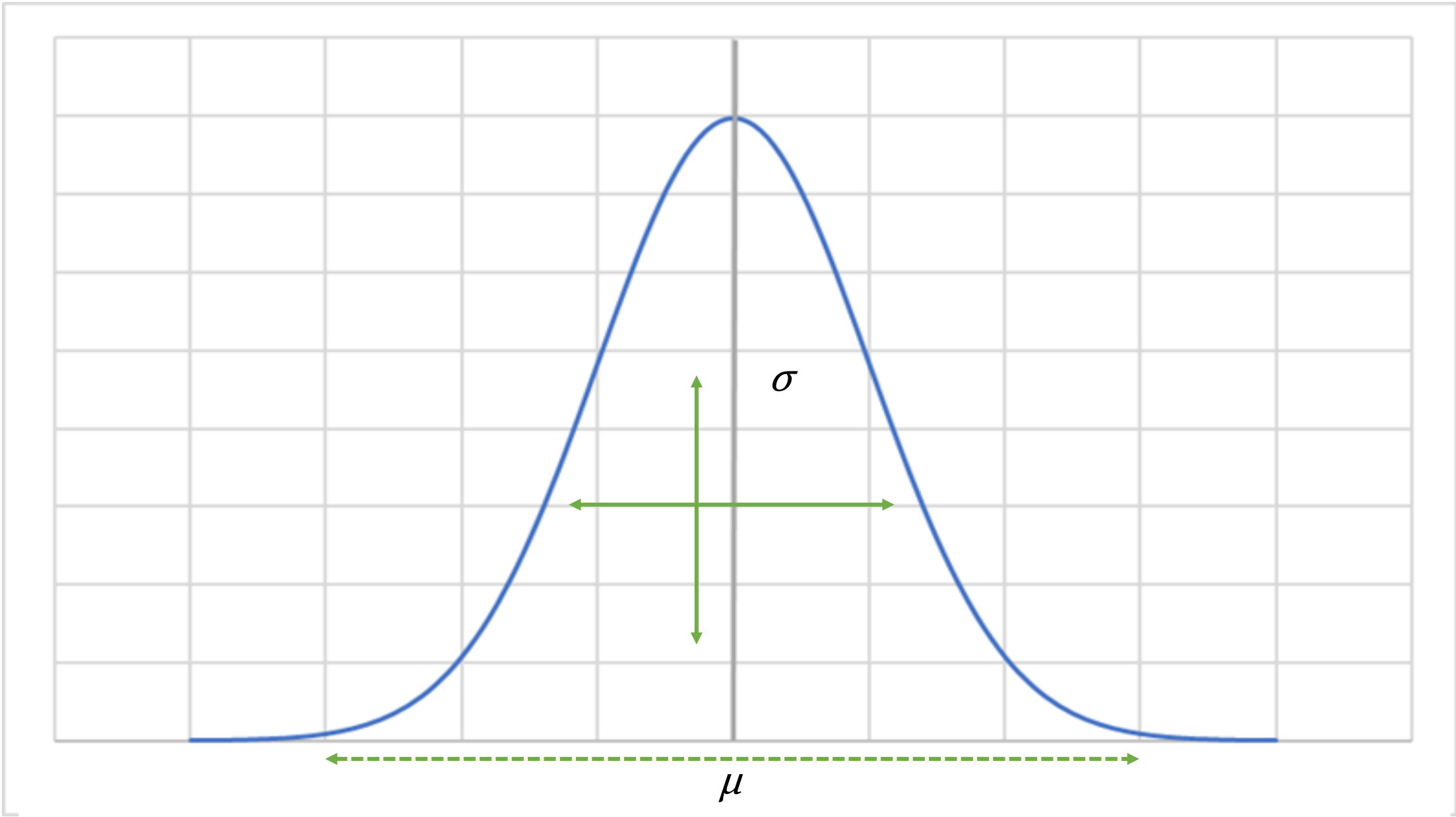
Normal distribution

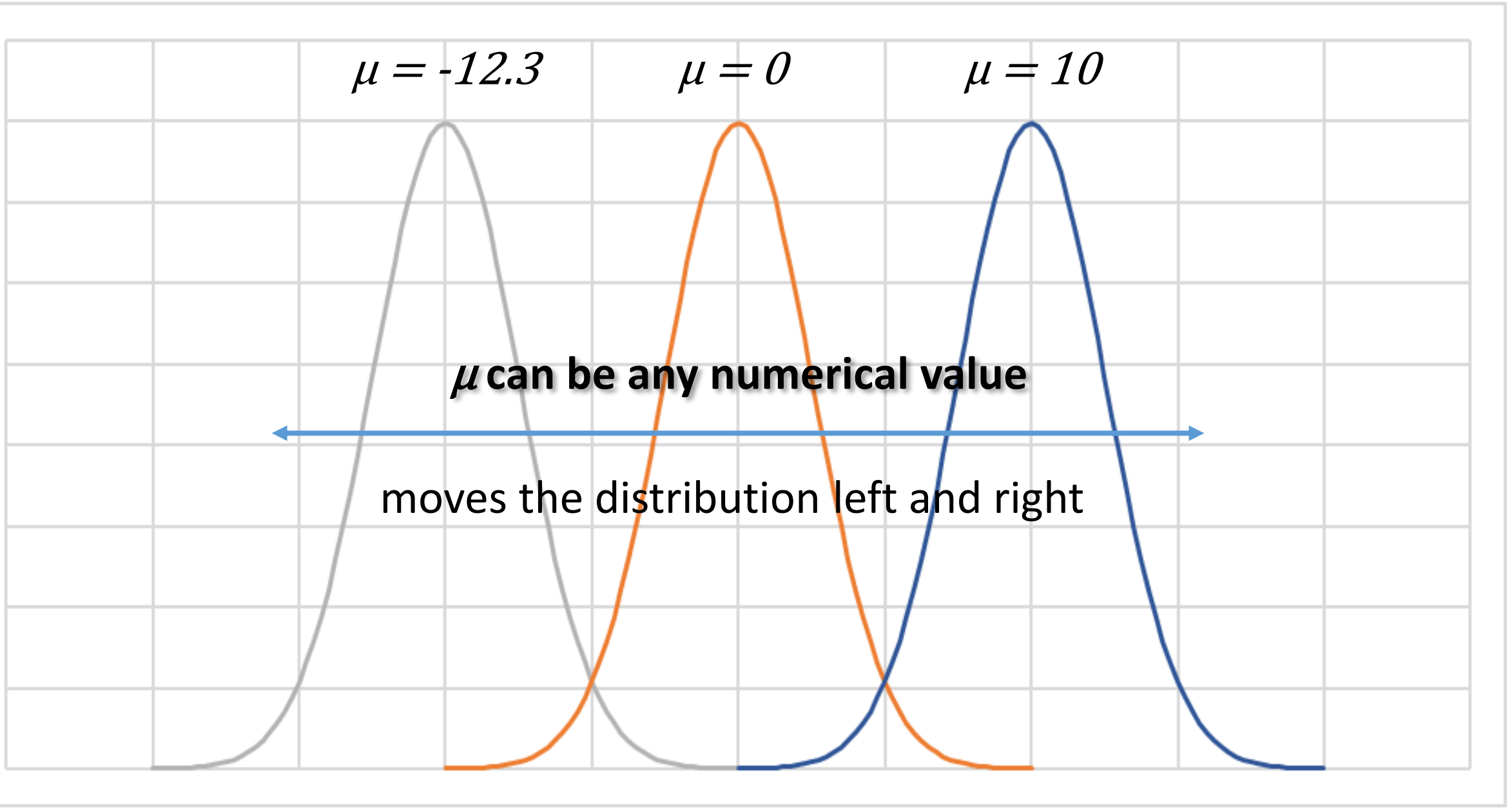
- Oftentimes data is described as being “normal” (in the statistical sense). But what does that mean?
- Let's start by discussing the frequency with which various events, whether natural or man-made, occur
  - Natural: human height, body temperatures, blood pressure etc.
  - Man-made: Financial data, sales etc.
- For these measures, the mean tends to be very frequent while measures away from the mean are less frequent
- Let's take a look at the normal distribution to learn more about its properties

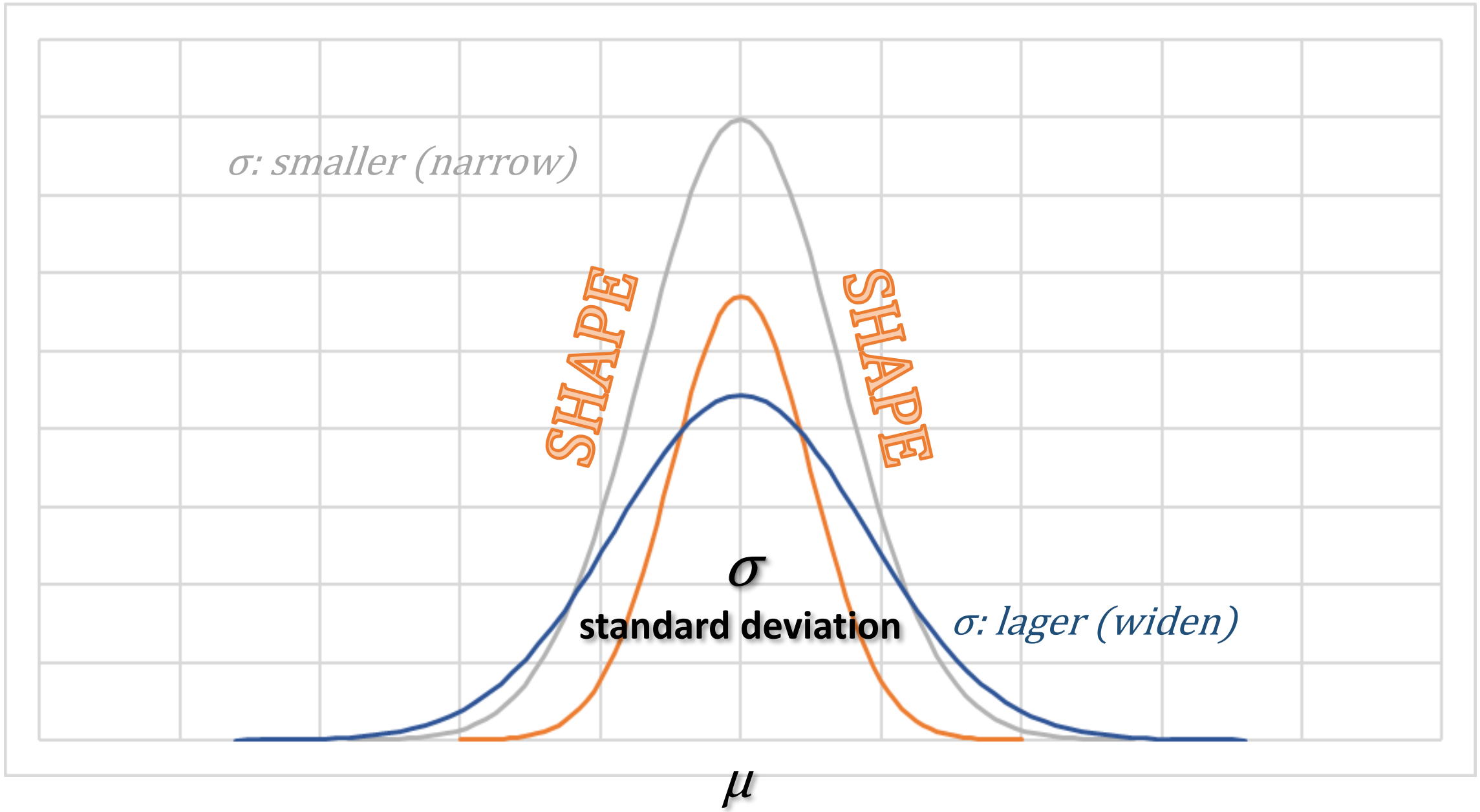

Probability area
Lower tail
Upper tail
-5
-4
-3
-2
-1
0
1
2
3
4
5


Symmetry
-5
-4
-3
-2
-1
0
1
2
3
4
5


mean, median, mode
-5
-4
-3
-2
-1
0
1
2
3
4
5


σ
μ


$\mu = -12.3$
$\mu = 0$
$\mu = 10$
μ can be any numerical value
moves the distribution left and right


σ: smaller (narrow)
SHAPE
SHAPE
σ
standard deviation
σ: lager (widen)
μ

The Standard Normal Curve

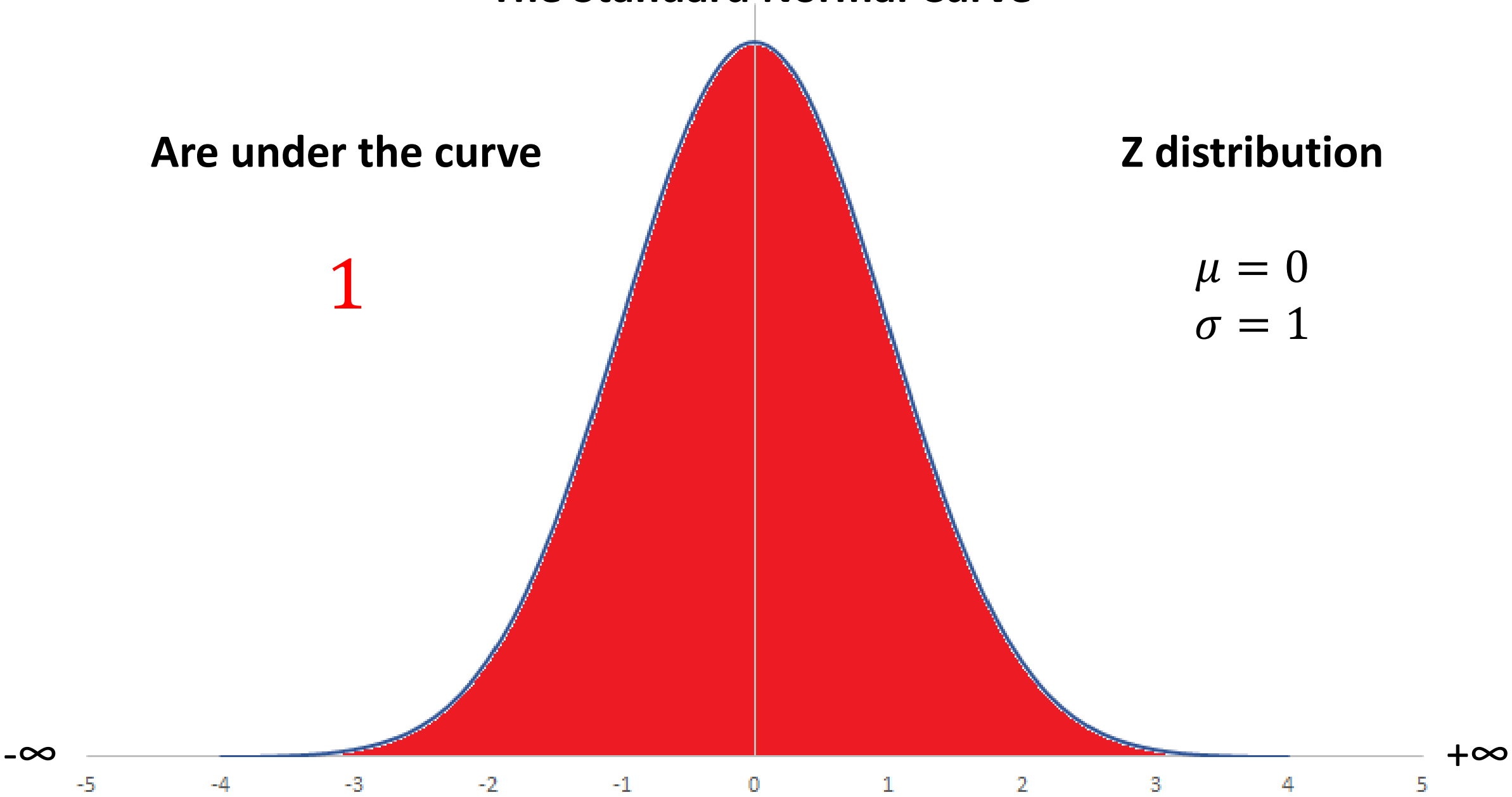
Are under the curve

1

Z distribution

$\mu = 0$

$\sigma = 1$



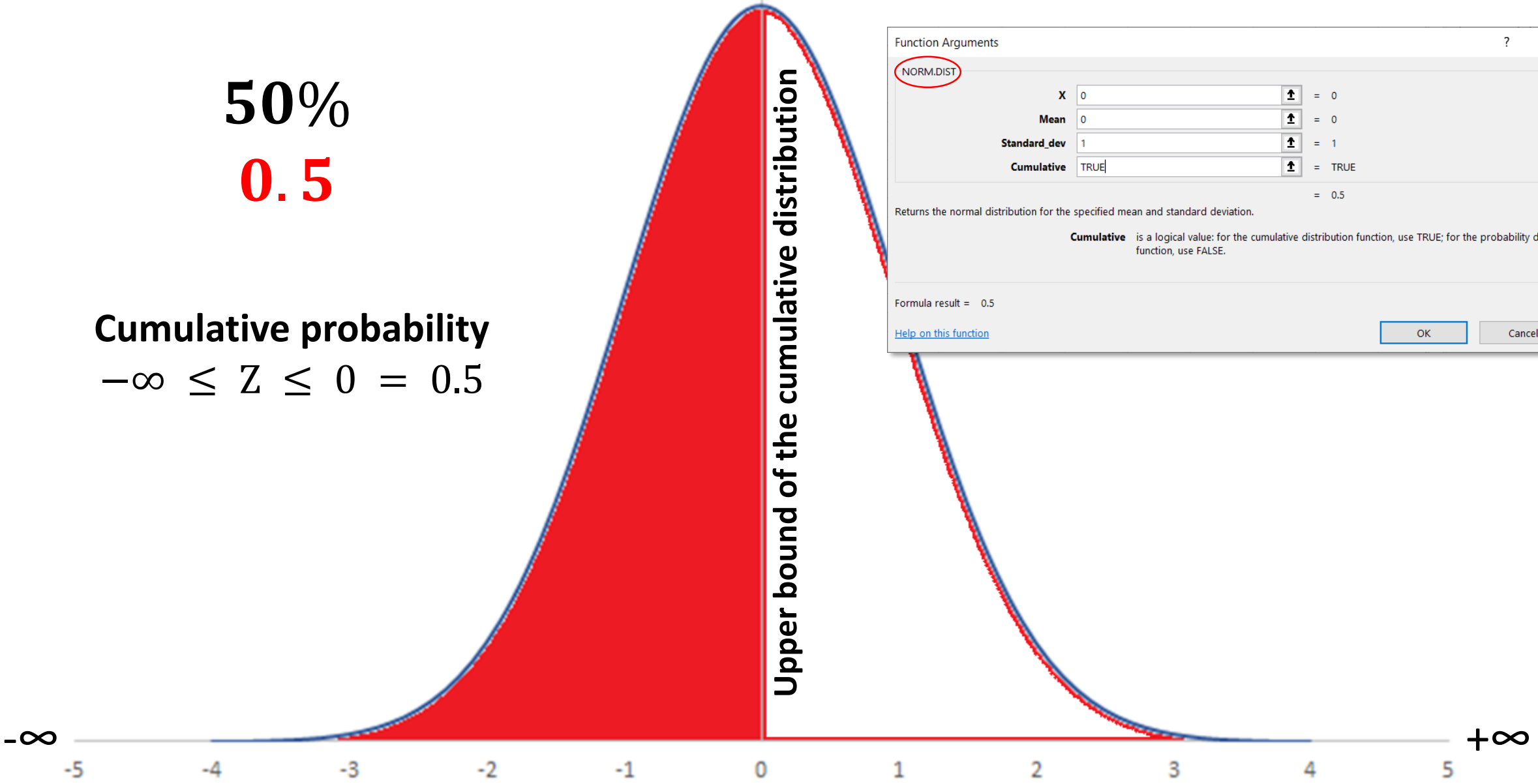
The Standard Normal Curve

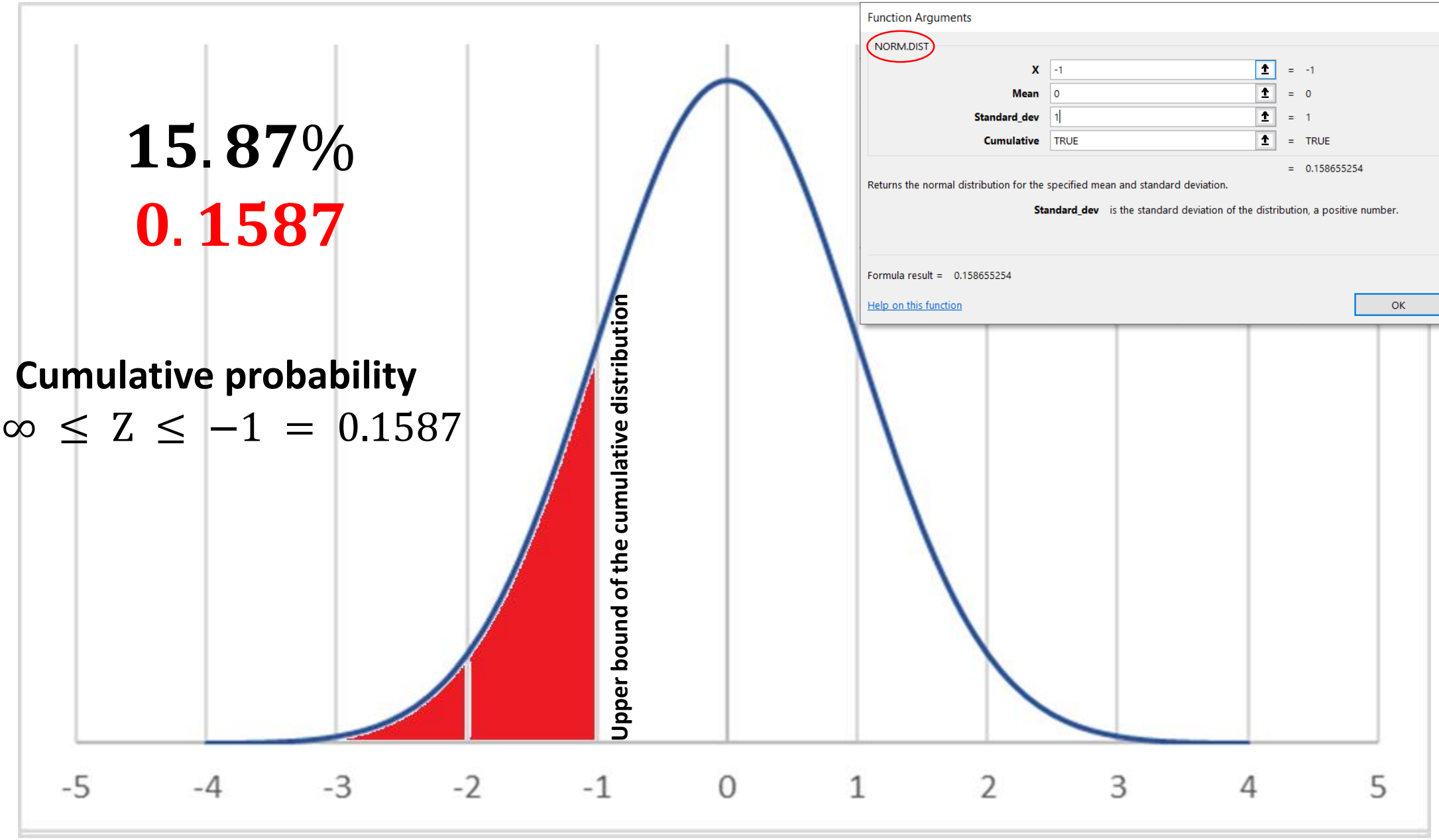
50%

0.5

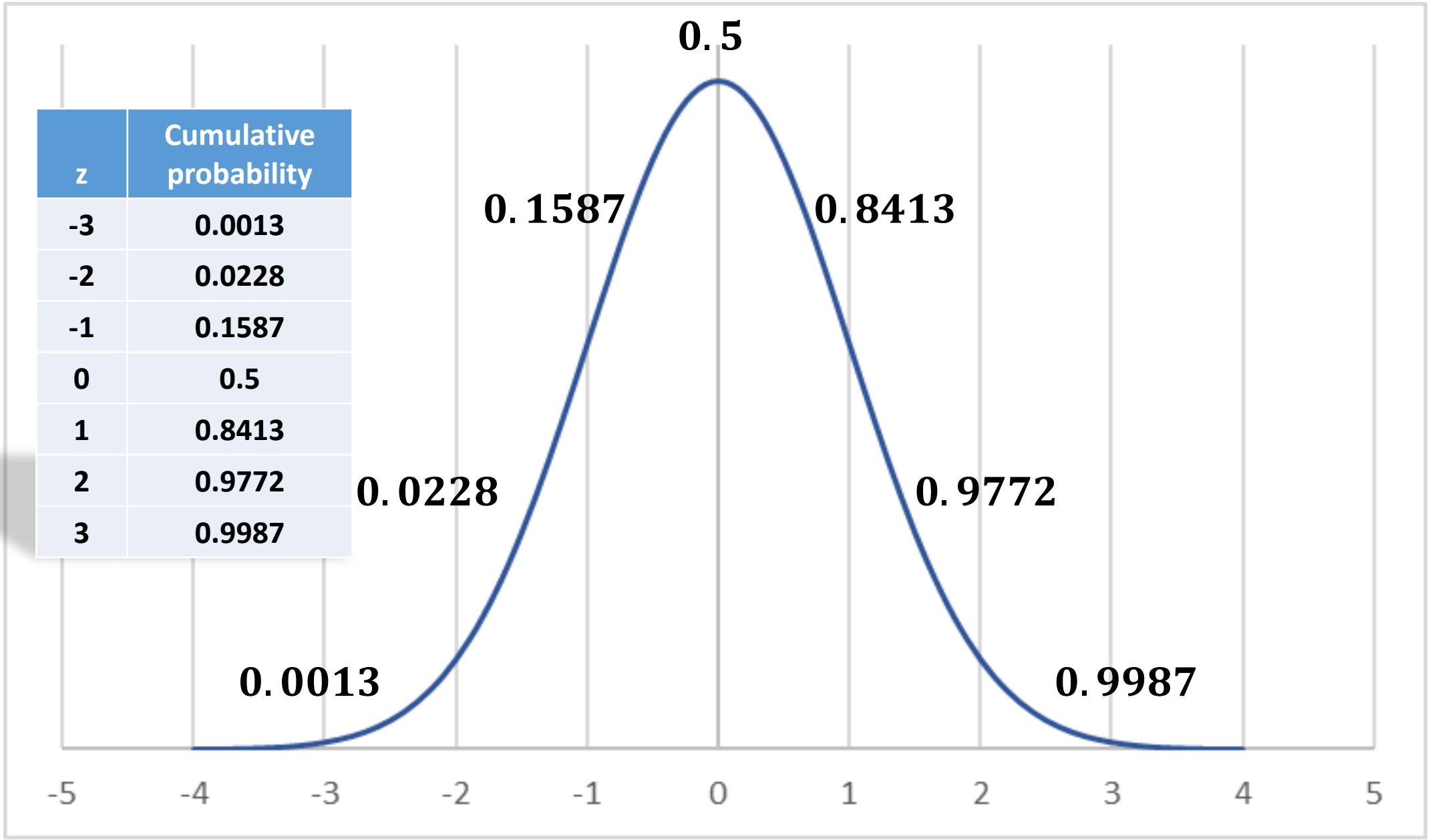
Cumulative probability

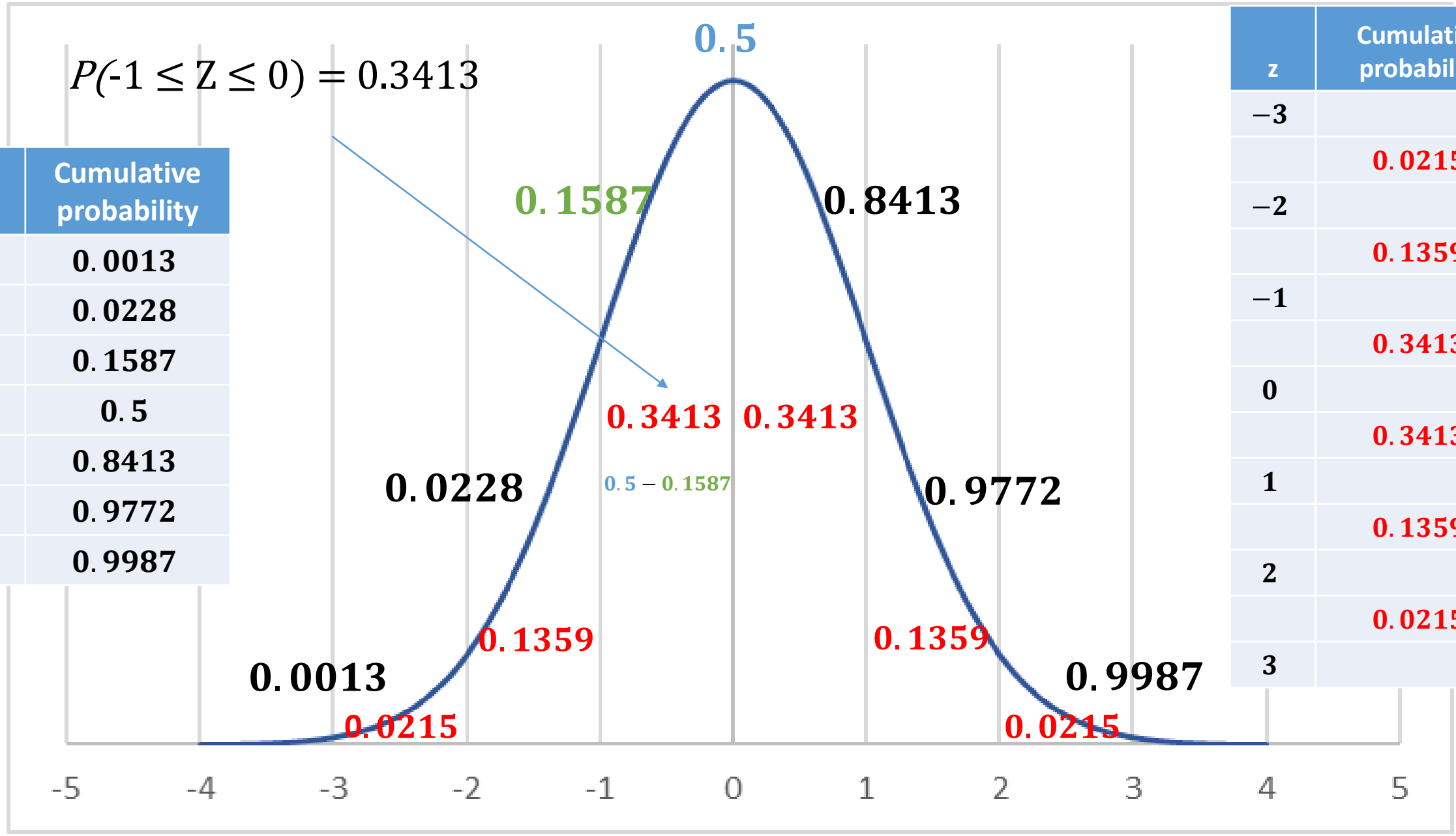
$$-\infty \leq Z \leq 0 = 0.5$$




15.87%
0.1587
Cumulative probability
$-\infty \leq Z \leq -1 = 0.1587$
Upper bound of the cumulative distribution
-5
-4
-3
-2
-1
0
1
2
3
4
5
Function Arguments
NORM.DIST
X
-1
= -1
Mean
0
= 0
Standard_dev
1
= 1
Cumulative
TRUE
= TRUE
= 0.158655254
Returns the normal distribution for the specified mean and standard deviation.
Standard_dev is the standard deviation of the distribution, a positive number.
Formula result = 0.158655254
Help on this function
OK
Cancel

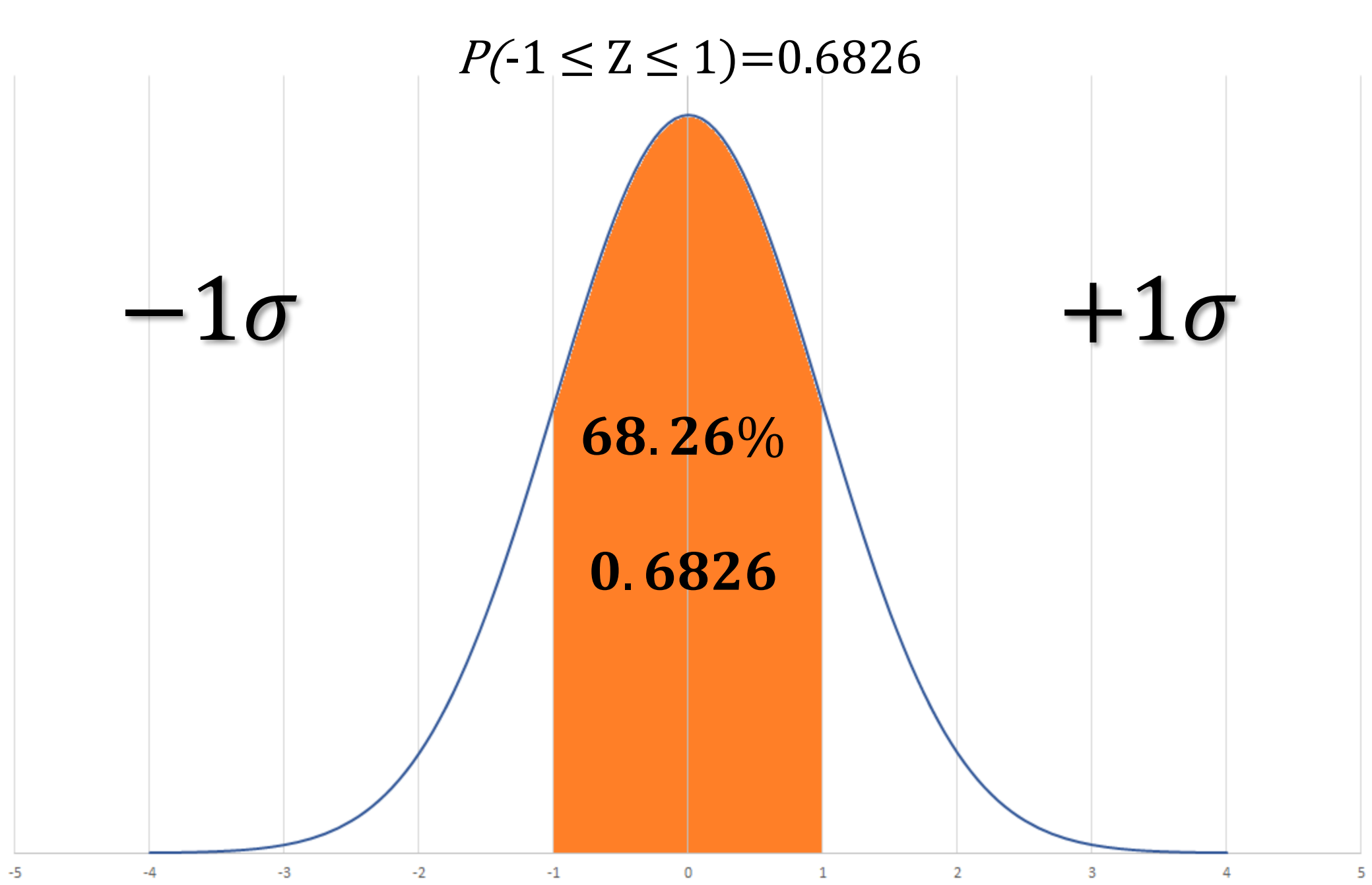
z	Cumulative probability
-3	0.0013
-2	0.0228
-1	0.1587
0	0.5
1	0.8413
2	0.9772
3	0.9987



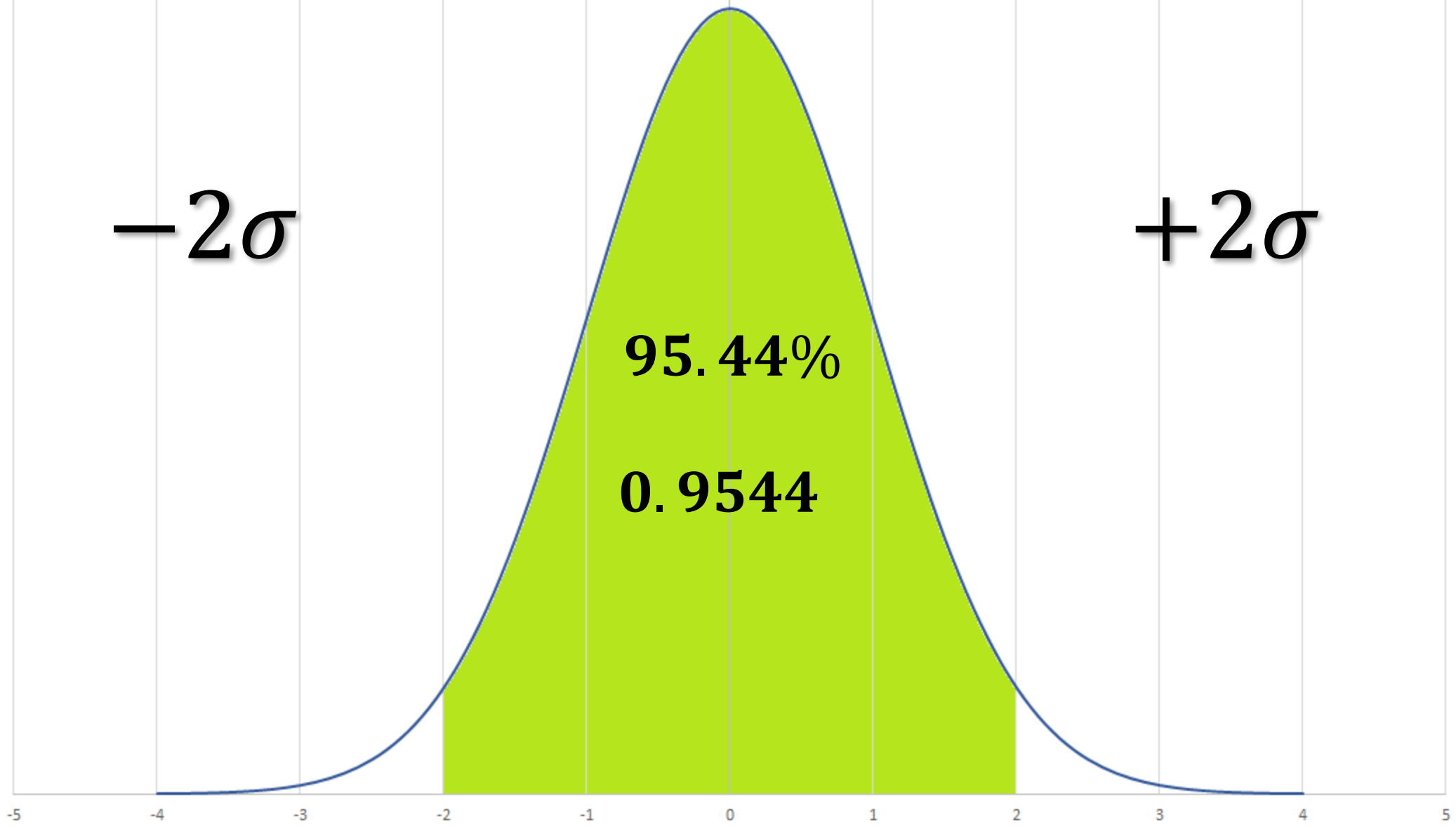
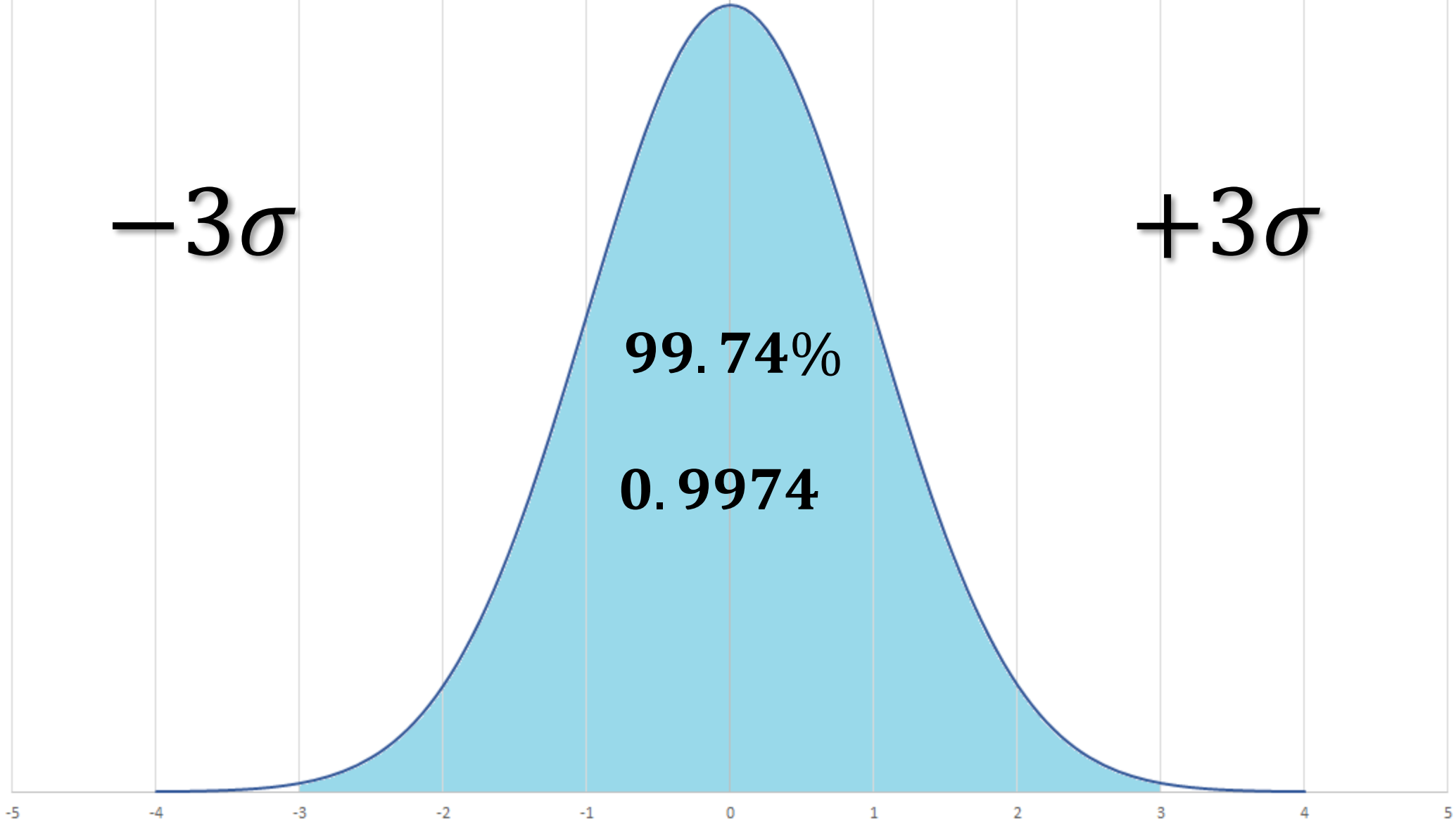


z	Cumulative probability
−3	0.0013
−2	0.0228
−1	0.1587
0	0.5
1	0.8413
2	0.9772
3	0.9987

z	Cumulative probability
−3	
	0.0215
−2	
	0.1359
−1	
	0.3413
0	
	0.3413
1	
	0.1359
2	
	0.0215
3	


$P(-1 \leq Z \leq 1)=0.6826$
-1σ
$+1\sigma$
68.26%
0.6826
-5
-4
-3
-2
-1
0
1
2
3
4
5

$P(-2 \leq Z \leq 2) = 0.9544$



$P(-3 \leq Z \leq 3) = 0.9974$