Paired samples t-test



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Evaluating the Effectiveness of Hypotensive Drug



Evaluating the Effectiveness of Hypotensive Drug

Twelve people were given a drug believed to have hypotensive properties. The systolic blood pressure of these subjects, measured in millimeters of mercury, before and after drug administration, is shown on the next slide. We want to:

- a. statistically evaluate the potential hypotensive activity of the drug and
- b. establish the reliability limits of the mean hypotensive activity



Entering data and defining variables

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Running the Paired-Samples T Test

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 To analyze the data, select Analyze -> Compare Means -> Paired-Samples T Test... (1)

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2. Drag the two variables from the left panel to the **Paired Variables** box, and press **OK**

Results and interpretation



- The drug indeed possesses hypotensive effects (p < 0.05, statistically significant)
- Mean hypotensive response was
- estimated to be 13.0 mm Hg
 - With a 95% confidence internal
 (CI), the true mean hypotensive
 response lies between 4.57 και
 21.43. Since 0 is not included in
 the 95% CI, we conclude that the
 mean response is statistically
 significant

Mean difference = 148.42 - 135.42 = 13 t = (Mean difference)/SE=13/3.83=3.39

Change in the Amount of Hormone in the Blood



Change in the Amount of Hormone in the Blood

Before	After
4.30	29.60
4.60	25.10
5.20	15.50
5.20	29.60
6.60	24.10
7.20	37.80
8.40	20.20
9.00	21.90
10.40	14.20
14.00	34.60
17.80	46.20

The amount of a hormone found in the blood of eleven patients before and after treatment administration is provided. Did the treatment change the patients' average hormone levels?

Null hypothesis (H₀): There is no significant difference in hormone levels before and after treatment administration



Entering data and defining variables

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6	7.20	37.80										
7	8.40	20.20										
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Running the Paired-Samples T Test



 To analyze the data, select Analyze -> Compare Means -> Paired-Samples T Test... (1)

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2. Drag the two variables from the left panel to the **Paired Variables** box, and press **OK** (2)

Result and interpretation



- The treatment significantly increases the hormone levels in the blood (t(10)=-7.46, p<0.001)
 - The mean increase in hormone levels was estimated to be 18.74 units

df

10

+

-7.460

Sig. (2-tailed)

.000

With 95% confidence interval (CI), the true increase lies between 13.14 and 24.33. Since 0 is not included in the 95% confidence interval (CI), we conclude that the increase is statistically significant

Hemoglobin change after administration of EPO



Hemoglobin change after administration of EPO

Suppose we want to compare the hemoglobin change after administration of erythropoietin (EPO) in 9 patients. The data (g/L) are as follows::

Subject	After use of EPO	Before
1	160	135
2	157	126
3	153	165
4	165	122
5	155	162
6	160	122
7	165	116
8	170	136
9	157	168

Null hypothesis (H_0) : There is no change in hemoglobin levels in the patients before and after erythropoietin administration.

Entering data and defining variables

We enter the data in the **Data View** and define the variables in the **Variable View** as shown below

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Since these are **paired observations**, the normality test will be conducted on the difference between them. To compute this, select **Transform -> Compute Variable** from the menu. In the window that appears (2), drag the before variable into the **Numeric Expression:** box, type a minus sign, then drag the after_epo variable into the same box. Enter the name diff for the new variable in the **Target Variable:** field and click **OK**.

Transform Analyze Direct Marketing Graphs Compute Variable.. Programmability Transformation... Count Values within Cases... Shift Values... 🔤 Recode into Same Variables... Recode into Different Variables... Automatic Recode... Create Dummy Variables Visual Binning... Rank Cases... 🚍 Date and Time Wizard... 🚾 Create Time Series.. 📲 Replace Missing <u>V</u>alues... 🍘 Random Number <u>G</u>enerators.. Run Pending Transforms Ctrl+G





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3	165	153	12,00
4	122	165	-43,00
5	162	155	7,00
6	122	160	-38,00
7	116	165	-49,00
8	136	170	-34,00
9	168	157	11,00



Next, select **Analyze -> Descriptive Statistics -> Explorer** from the menu. In the window that appears (2) drug the new **diff** variable from the left box to the **Dependent List:** box. Then, choose **Plots...** ...

Analyze Direct Marketing Graphs	: <u>U</u> t	ilities E <u>x</u> tensions <u>W</u>	1	Explore			×		
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Spatial and Temporal Modeling	۲			Histrogram and	Norr	nality plots with	tests.	<u>Continue</u> Cancel	Help

Press Continue and OK.

Among other results, the following appear in the SPSS output window.

	Case Processing Summary												
	Cases												
	Va	lid	Miss	sing	To	tal							
	N	Percent	N	Percent									
diff	9	100.0%	0	0.0%	9	100.0%							

Tests of Normality							
	Kolmogorov-Smirnov ^a				Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.	
diff	.230	9	.186	.843	9	.062	
ali	illiefors Signif	ficance Corr	ection				

Based on the **Shapiro** – **Wilk** test (n < 50), we conclude that the differences in hemoglobin values follow a normal distribution, as the p-value (Sig.) is greater than 0.05 (p-value = 0.062).

Note: The test of normality **Kolmogorov-Smirnov** is used when the number of patients exceeds 50.

Descriptives								
			Statistic	Std. Error				
diff	Mean	-21.1111	8.11282					
-	95% Confidence Interval	Lower Bound	-39.8193					
	for Mean	Upper Bound	Jpper Bound -2.4029					
	5% Trimmed Mean	-21.4012						
	Median	-31.0000						
	Variance	592.361						
	Std. Deviation	24.33847						
	Minimum	-49.00						
	Maximum	12.00						
	Range	61.00						
	Interquartile Range	49.50						
	Skewness	.574	.717					
	Kurtosis		-1.638	1.400				

Test of normality using Q-Q plots

The figure (1) below shows the **Q-Q plot** of the differences in hemoglobin values, indicating that these values follow **a normal distribution well**



Test of normality using histograms

Figures 1 shows the **histogram** of the difference in the hemoglobin values, indicating that they **do not follow a normal distribution** (note that histograms can sometimes be misleading)



The **normality curve** was displayed by doubleclicking on the histogram and selecting the **Show Distribution Curve** icon from the toolbar



Running the Paired-Samples T Test



To analyze the data, select Analyze -> Compare Means -> Paired-Samples T Test... (1)

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OK Paste Reset Cancel Help								

2. Drag the two variables from the left panel to the **Paired Variables** box, and press **OK**

Results and Interpretation

		Mean	N	Std. Deviati	Std. Error on Mean					
air 1	before	139,11		9 20,4	54 6,8	18				
	after_epo	160,22		9 5,4	95 1,8	32				
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Pair 1	before & aft	er_epo	9	-,640	,064					
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			95% Confidence Interval of t Std. Error Difference		e Interval of the ence			Ļ		
			Mean	Std. Deviation	Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	before - afte	er_epo	-21,111	24,338	8,113	-39,819	-2,403	-2,602	8	,032



- The administration of EPO indeed increased the hemoglobin levels
- The mean difference (21.111 g/L) in hemoglobin levels was found to be statistically significant at p < 0.05
 - With 95% confidence interval (CI), the true mean difference lies between -39.819 and -2.403. Since 0 is not included in the 95% confidence interval (CI), we conclude that the mean difference is statistically significant