Z-test

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Quick Reviews

t-Tests

- **Purpose**: Used to **compare the means** between **two groups**
- **Types**:
  - **Independent samples t-test**: Compares means of two independent groups
  - **Paired samples t-test**: Compares means of the same group at two different times or conditions
- **Assumptions**: Normal distribution, similar variances (homogeneity), independent observations
- **Example Use**: Testing if a new drug leads to different average blood pressure compared to a placebo

ANOVA (Analysis of Variance)

- **Purpose**: Used to compare means across three or more groups.
- **Types**:
  - **One-Way ANOVA**: Examines the effect of one independent variable (factor) on the dependent variable
  - **Two-Way ANOVA**: Examines the effect of two independent variables and their interaction on the dependent variable
- **Assumptions**: Normal distribution, homogeneity of variances, independent observations
- **Example Use**: Testing the effect of different diets on weight loss (One-Way ANOVA) or testing the effect of diet and exercise type on weight loss (Two-Way ANOVA).

Evaluating Asthma Rates: Hospital Sample Versus General Population

One Population Proportion Z-Test

Assumptions

- Random Sampling
  - Each sample should be drawn randomly from its respective population
- Large Enough Sample Size (Normality Assumption)
  - $n \geq 30$
- Binary Outcome (Success/Failure)
  - For example, in the context of asthma rates, each individual in the sample either has a history of asthma (success) or does not have a history of asthma (failure)

Evaluating Asthma Rates: Hospital Sample Versus General Population

Example

In a hospital, a random sample of $n_1 = 215$ women was collected from patient lists, and $r = 39$ of them were found to have a history of asthma (i.e., the observed rate of asthma is $p = \frac{39}{215} = 18\%$). It is known that the prevalence rate of the disease is $P = 15\%$

Question

Does the percentage of women diagnosed with asthma in the sample match the prevalence in the general population?

Hypothesis

$\boldsymbol{H_0}$: The asthma rate in the hospital sample is equal to the prevalence rate in the general population

$$H_0 : p = P$$

$\boldsymbol{H_\alpha}$: The asthma rate in the hospital sample is different from the prevalence rate in the general population

$$H_\alpha : p \neq P$$

Z-test formula

To determine if the percentage of cases in the sample differs from the disease prevalence in the general population, we use the z-test

The formula for the z-test statistic is:

$$z = \frac{p - P}{SE}$$

$$SE = \sqrt{\frac{P(1 - P)}{n}}$$

Step 1: Find the sample proportion

$$p = \frac{r}{n_1} = \frac{39}{215} = 0.1814$$ that is 18.14%

$$P = 15\%$$

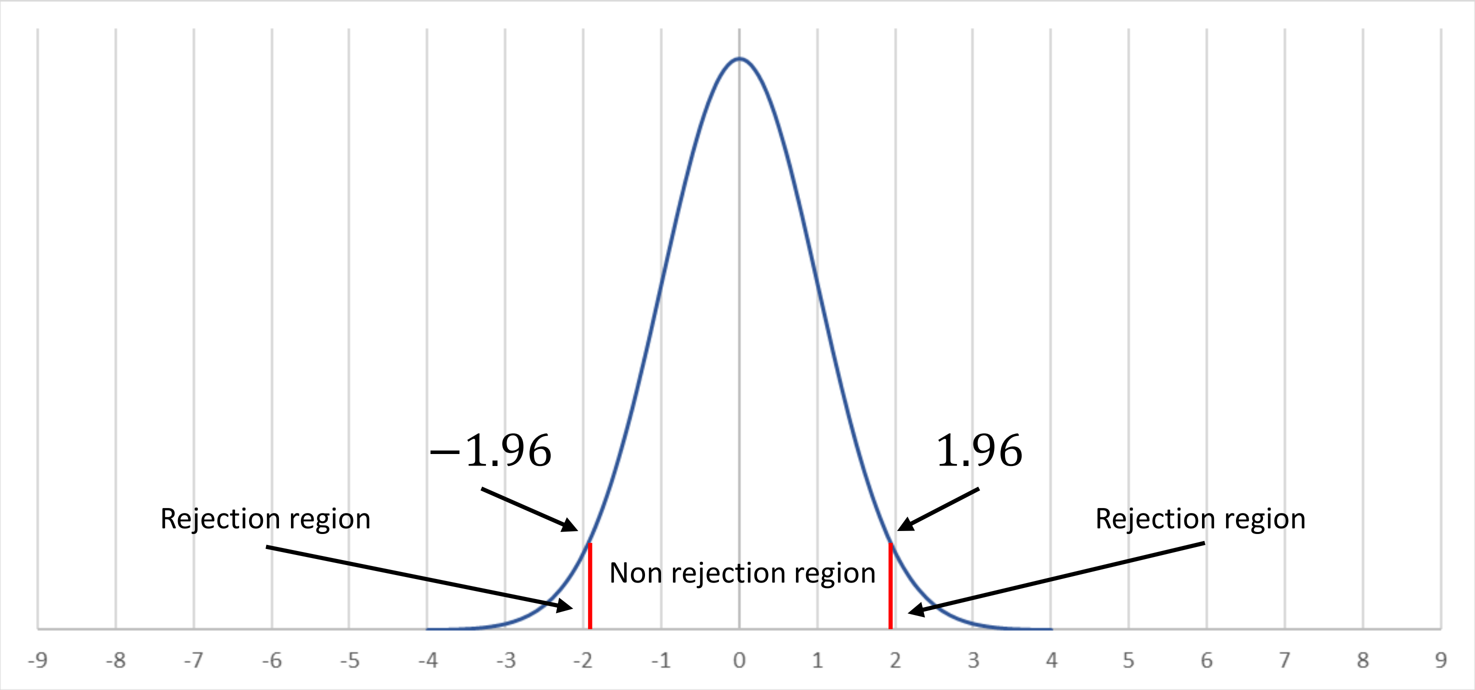
Step 2: Inserting the numbers from Step 1 into the test statistic formula

$$SE = \sqrt{\frac{0.15(1-0.15)}{215}} = 0.0244$$

$$z = \frac{0.1814 - 0.15}{0.0244} = 1.2892$$

We must determine whether the z-score is within the "rejection region".

Step 3: z-score associated with a 5% alpha level

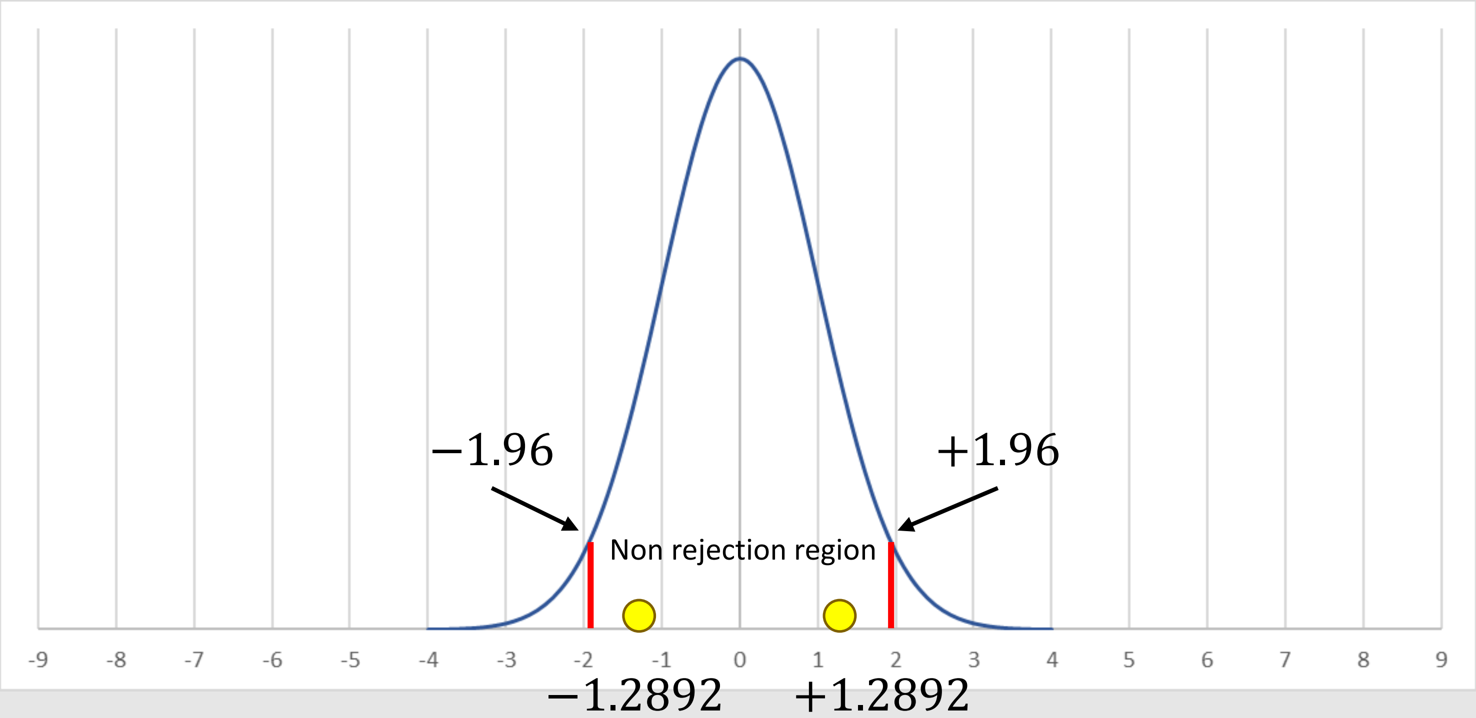


Step 4: Statistical significance

The value $z = 1.2892$ is less than the 5% critical value of the standard normal distribution, which is 1.96

The results ($P \geq \ 0.05$) of the z-test indicated that the observed rate of asthma (18.14%) did **not significantly differ** from the population prevalence rate (15%)

This lack of statistical significance suggests that the increased prevalence observed in the sample does not provide sufficient evidence to assert that the rate of asthma among the women in this study is different from that of the general population



95% confidence interval

The 95% confidence interval (CI) is given by the following formula:

$$(p - 1.96 \cdot SE, p + 1.96 \cdot SE)$$

$$SE = \sqrt{\frac{P(1-P)}{n}}$$

95% confidence interval

$$(0.1814 - 1.96 \cdot 0.0244, 0.1814 + 1.96 \cdot 0.0244)$$

$$(0.1814 - 0.047824, 0.1814 + 0.047824)$$

$$(0.133576{,}0.229224)$$

$$(0.1336, 0.2292)$$

95% confidence interval

The 95% confidence interval (CI) for the asthma prevalence rate among the sample of women is

$$(0.1336, 0.2292)$$

If we assume that the sample of women from the hospital is representative of all women in the country, we can say that we are we are 95% confident that the true asthma prevalence rate for the population of interest falls between 13.36% and 22.92%

Since 15% falls within the CI $(13.36\%, 22.92\%)$, we conclude that the sample prevalence is **not significantly different** from the population prevalence at the 5% level

Z-Test with continuity correction

For relatively small samples, the following z-test gives more accurate results:

$$z_c = \frac{|p - P| - \frac{1}{2n}}{SE}$$

$$z_c = \frac{|0.1814 - 0.15| - \frac{1}{2 \cdot 215}}{0.0244}$$

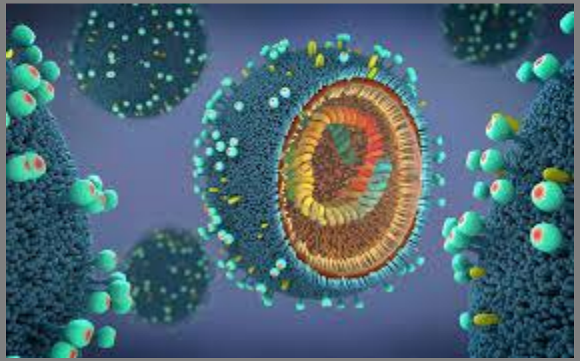
In our example, $z_c = 1.1937$ is slightly smaller than $z = 1.2892$ (which we found without the correction) because the sample size is large

Practice

1. A study of diseases in native Americans (Kizer et al. 2006) found 381 obese or overweight patients in 449 patients. In the general population of the USA, the percentage obese or overweight is 65%.
   The researchers wanted to determine if the percentage of obesity/overweight native Americans was different than that of the general population
2. Kim et al. (2004) studied the measles-rubella vaccination-rates in Korea. They compared the proportion of children with measles antibodies to the World Health Organization (WHO) target proportion (for children aged 5 to 9 years old: 10%)
   The aim of the study was to test if the proportion of Korean children with the measles antibody in the population was 10% or lower (i.e., better). In the study, 55 children out of 972 had the antibody present

Comparing Influenza Rates in Vaccinated vs. Placebo Groups

Two Proportion Z-Test



Assumptions

- Independent Samples
  - The two samples should be independent of each other, meaning that the selection of individuals in one sample does not influence the selection in the other
- Random Sampling
  - Each sample should be drawn randomly from its respective population
- Large Enough Sample Size (Normality Assumption)
  - $n \geq 30$
- Binary Outcome (Success/Failure)
  - For example, in the context of influenza rates, each individual in the sample either got influenza (success) or did not get influenza (failure)

Comparing Influenza Rates in Vaccinated vs. Placebo Groups

Example

Of the 240 ($n_1 = 240$) people vaccinated with the real vaccine, 20 ($p_1 = 20$) got influenza, compared to 80 ($p_2 = 80$) out of 220 ($n_2 = 220$) who were vaccinated with a placebo.

Question

Is there any indication that the vaccine was effective?

Hypothesis

$\boldsymbol{H_0}$: The vaccination does not affect the influenza rate; the proportions of influenza cases in the vaccinated group (p_1) and the placebo group (p_2) are equal

$$H_0: p_1 = p_2$$

$\boldsymbol{H_\alpha}$: The vaccination does affect the influenza rate; the proportions of influenza cases in the vaccinated group (p_1) and the placebo group (p_2) are not equal

$$H_\alpha: p_1 \neq p_2$$

Z-test formula

To test whether the flu vaccination rate differs from the placebo rate, a z-test is used.

The formula for the z-test statistic is:

$$z = \frac{p_1 - p_2}{SE}$$

$$SE = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Step 1: Find the two proportions

$$p_1 = \frac{p_1}{n_1} = \frac{20}{240} = 0.083 \qquad \text{that is } 8.3\%$$

$$p_2 = \frac{p_2}{n_2} = \frac{80}{220} = 0.364 \qquad \text{that is } 36.4\%$$

Step 2: Find the overall sample proportion

$$p = \frac{p_1 + p_2}{n_1 + n_2}$$

$$p = \frac{20 + 80}{240 + 220}$$

$$p = 0.217$$

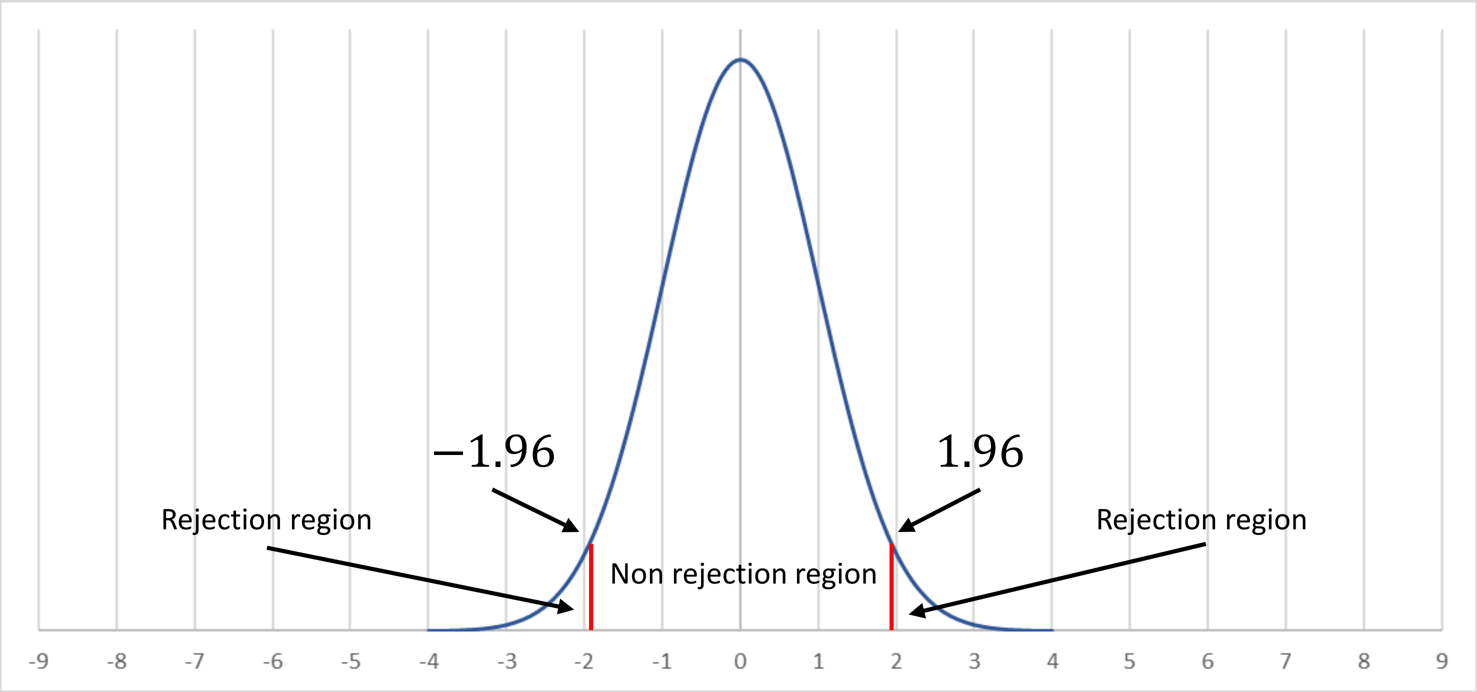
Step 3: Inserting the numbers from Step 1 and Step 2 into the test statistic formula

$$SE = \sqrt{0.217(1-0.217)\left(\frac{1}{240}+\frac{1}{220}\right)} = 0.038$$

$$z = \frac{0.083 - 0.364}{0.038} = -7.30$$

We must determine whether the z-score is within the "rejection region".

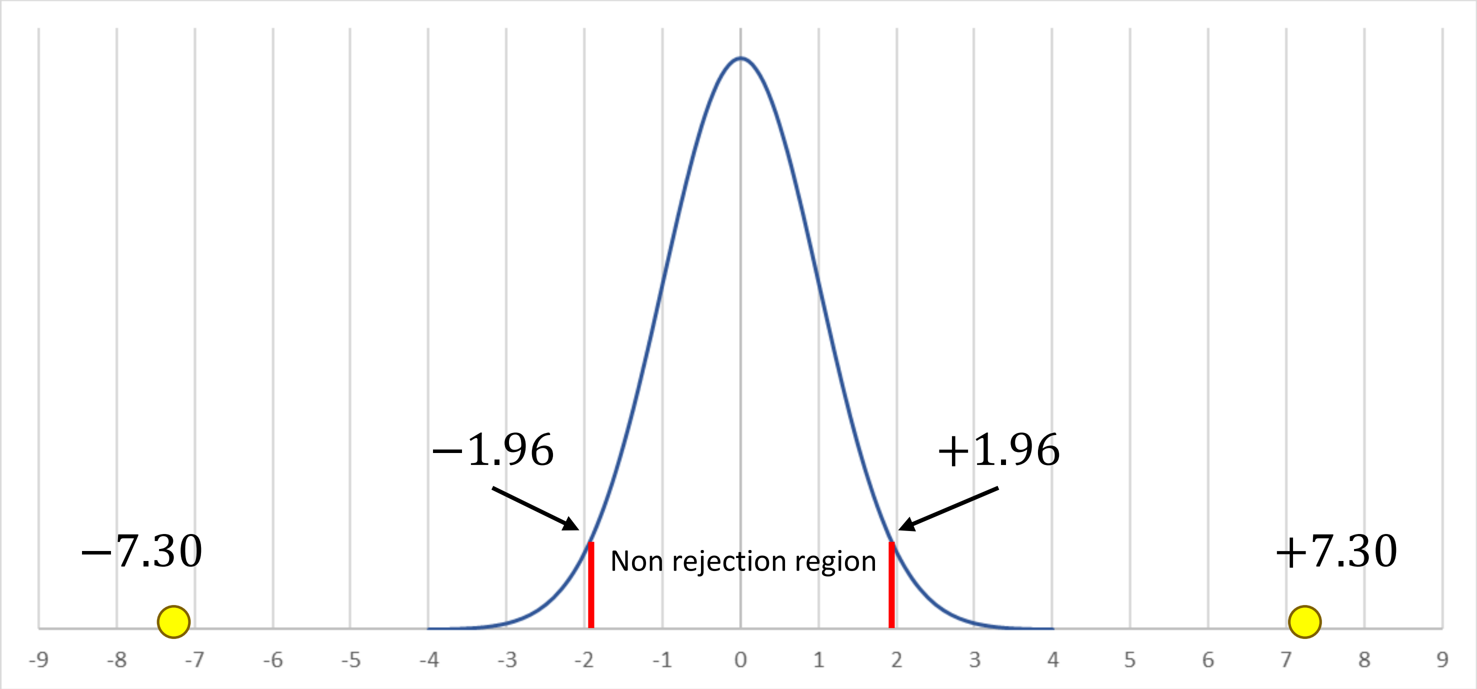
Step 4: z-score associated with a 5% alpha level



Step 5: Statistical significance

The value $z = -7.30$ is less than the 5% critical value of the standard normal distribution, which is -1.96

Therefore, there is evidence ($P < 0.05$) that actual vaccination reduces the risk of contracting influenza, **statistically significant**



95% confidence interval

The 95% confidence interval (CI) is given by the following formula:

$$\left((p_1 - p_2) - 1.96 \cdot SE, (p_1 - p_2) + 1.96 \cdot SE\right)$$

$$SE = \sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}}$$

95% confidence interval

$$\left((p_1 - p_2) - 1.96 \cdot SE, (p_1 - p_2) + 1.96 \cdot SE\right)$$

$$\left((0.083 - 0.364) - 1.96 \cdot 0.038, (p_1 - p_2) + 1.96 \cdot 0.038\right)$$

$$(-0.281 - 0.07448, -0.281 + 0.07448)$$

$$(-0.35548, -0.20652)$$

$$(-0.3555, -0.2065)$$

95% confidence interval

The 95% confidence interval for the reduction in the flu rate due to the vaccine is

$$(-0.3555, -0.2065)$$

Therefore, the risk of contracting the flu when vaccinated is between 20.65% and 35.55% lower than when given a placebo

The fact that the 95% confidence interval **does not include 0** suggests that the difference between the two proportions is **statistically significant at the 5% level**

Practice

1. Researchers want to test the effectiveness of a new anti-anxiety medication. In clinical testing, 64 out of 200 people taking the medication report symptoms of anxiety. Of the people receiving a placebo, 92 out of 200 report symptoms of anxiety. Is the medication working any differently than the placebo? Test this claim using alpha = 0.05.

2. Suppose a Drug Company develops a new drug, designed to prevent colds. The company states that the drug is equally effective for men and women. To test this claim, they choose a simple random sample of 100 women and 200 men from a population of 100.000 volunteers.

   At the end of the study, 38% of the women caught a cold; and 51% of the men caught a cold. Based on these findings, can we reject the company's claim that the drug is equally effective for men and women? Use a 0.05 level of significance.